Advanced Courses in Mathematics CRM Barcelona

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# Stochastic Integration by Parts and Functional Itô Calculus

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Rama Cont dedicates his contribution to Atossa, for her kindness and constant encouragement.

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# Foreword

During July 23th to 27th, 2012, the first session of the *Barcelona Summer School on Stochastic Analysis* was organized at the Centre de Recerca Matemàtica (CRM) in Bellaterra, Barcelona (Spain). This volume contains the lecture notes of the two courses given at the school by Vlad Bally and Rama Cont.

The notes of the course by Vlad Bally are co-authored with her collaborator Lucia Caramellino. They develop integration by parts formulas in an abstract setting, extending Malliavin's work on abstract Wiener spaces, and thereby being applicable to prove absolute continuity for a broad class of random vectors. Properties like regularity of the density, estimates of the tails, and approximation of densities in the total variation norm are considered. The last part of the notes is devoted to introducing a method to prove existence of density based on interpolation spaces. Examples either not covered by Malliavin's approach or requiring less regularity are in the scope of its applications.

Rama Cont's notes are on Functional Itô Calculus. This is a non-anticipative functional calculus extending the classical Itô calculus to path-dependent functionals of stochastic processes. In contrast to Malliavin Calculus, which leads to *anticipative* representation of functionals, with Functional Itô Calculus one obtains *non-anticipative* representations, which may be more natural in many applied problems. That calculus is first introduced using a pathwise approach (that is, without probabilities) based on a notion of directional derivative. Later, after the introduction of a probability on the space of paths, a weak functional calculus emerges that can be applied without regularity conditions on the functionals. Two applications are studied in depth; the representation of martingales formulas, and then a new class of path-dependent partial differential equations termed *functional Kolmogorov equations.*

We are deeply indebted to the authors for their valuable contributions. Warm thanks are due to the Centre de Recerca Matemàtica, for its invaluable support in the organization of the School, and to our colleagues, members of the Organizing Committee, Xavier Bardina and Marta Sanz-Solé. We extend our thanks to the following institutions: AGAUR (Generalitat de Catalunya) and Ministerio de Economía y Competitividad, for the financial support provided with the grants SGR 2009-01360, MTM 2009-08869 and MTM 2009-07203.

Frederic Utzet and Josep Vives

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