Journal of Financial Market Infrastructures **12**(2), 1–40
DOI: 10.21314/JFMI.2025.001

Research Paper

Skin in the game: risk analysis of central counterparties

Rama Cont[1] and Samim Ghamami[2,3,4]

[1]Mathematical Institute, University of Oxford, Andrew Wiles Building, Radcliffe Observatory Quarter, Woodstock Road, Oxford OX2 6GG, UK; email: rama.cont@maths.ox.ac.uk

[2]Division of Economic and Risk Analysis, US Securities and Exchange Commission, 100 F Street Northeast, Washington, DC 20549, USA; email: ghamamis@sec.gov

[3]Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012, USA; email: sg185@nyu.edu

[4]Center for Risk Management Research, University of California, Berkeley, 579 Evans Hall, Berkeley, CA 94720-3880, USA

(Received November 5, 2024; revised February 19, 2025; accepted February 21, 2025)

ABSTRACT

We introduce a quantitative framework to design the capital contribution of a central counterparty (CCP) to its default waterfall, known as CCP "skin in the game" (SITG). We show that, under inadequate SITG levels, nondefaulting members are more exposed to default losses than CCPs. The resulting risk management incentive distortions could be mitigated by using the proposed framework. Our analysis addresses investor- and member-owned CCPs; we also analyze multilayer and "monolayer" default waterfalls. The broader central clearing mandate of US Treasuries may take place under monolayer CCPs. Viewing the total size of SITG as the lower bound on CCP regulatory capital, the framework can be used to improve

Corresponding author: S. Ghamami

Print ISSN 2049-5404 | Online ISSN 2049-5412

capital regulation of investor-and member-owned CCPs. We also show that bank capital rules for CCP exposures may underestimate risk.

Keywords: financial regulation; central clearing; over-the-counter markets; the US Treasury market; risk management; skin in the game.

1 INTRODUCTION

The over-the-counter (OTC) derivatives market reform program launched in 2009 by the Group of Twenty (G20) following the 2007–9 global financial crisis (GFC) has drastically transformed these markets. Central clearing of standardized OTC derivatives was one of the main components of this reform program (Ghamami and Glasserman 2017). The Covid-19 crisis revealed that the secondary market for US Treasuries can become dysfunctional, in part due to the constraints on the capacity of dealers that intermediate in this market (Duffie 2020; Duffie *et al* 2023). In 2021, the Group of Thirty (G30) proposed a different reform program to strengthen the resilience of the US Treasury markets (Group of Thirty 2021). Broadening central clearing mandates in government securities markets is one of the main elements of the 2021 reform program.

The main potential benefits of central clearing are well known. It has the potential to reduce the level of interconnectedness in OTC markets and improve transparency. Central clearing can help mitigate counterparty credit risk[1] through multilateral netting (Cont and Kokholm 2014; Duffie and Zhu 2011; Garratt and Zimmerman 2015; Ghamami and Glasserman 2017), and when multilateral netting efficiencies dominate bilateral netting, the cost of collateral requirements could be reduced.[2] Central clearing may also reduce the pressure on intermediaries' balance sheets (Baranova *et al* 2023; Duffie 2020).

[1] In US Treasury markets, counterparty credit risk materializes mostly in the form of settlement failures (Ingber 2017). US Treasury market settlement failures rose significantly in March 2020 (Duffie 2020, Figure 13).

[2] Multilateral netting outperforms bilateral netting under certain conditions (Cont and Kokholm 2014). For instance, as the number of asset-class-specific central counterparties increases, and when bilateral netting across an increasing number of asset classes is permitted, bilateral netting can dominate multilateral netting. The overall netting efficiencies achieved by central clearing could also depend on specifics of client clearing models. Research on client clearing is scarce (Committee on Payments and Market Infrastructures–Board of the International Organization of Securities Commissions 2022; Ghamami *et al* 2023).

Central counterparties (CCPs) require effective governance, regulatory oversight and highly robust risk management frameworks. Otherwise, increased use of CCPs may create financial stability risks (Bernanke 2011; Dudley 2014; Tucker 2014). The failure of a systemically important CCP can be disastrous.[3] The right design and regulation of CCPs continue to generate debate among industry participants, government officials and the public. In 2019 and 2020 major buy-side and sell-side firms called for regulatory action to make clearing houses safer (JPMorgan Chase 2020). The industry paper included a number of recommendations, one of which was “requiring CCPs to make material contributions of their own capital to the default waterfall in two separate tranches”.

CCPs rely on their “default waterfall” to manage the pooled counterparty credit risk in centrally cleared markets (Cont 2015; Murphy 2017). The typical waterfall structure is multilayered and consists of collateral posted by clearing members in the form of risk-sensitive initial margin (IM) and contributions to a quasi-risk-based default fund (DF), also known as a guarantee fund. CCPs also often make equity capital contributions to the waterfall. These capital contributions are often referred to as “skin in the game” (SITG). Currently, the level of SITG held by CCPs is not risk-based.

When a member defaults, the CCP first uses the “defaulter pays” resources to cover losses. Potential remaining losses are “mutualized” among the CCP and surviving members. SITG and surviving members’ DF assets can be used in the loss mutualization process. SITG often comes into play twice: first, right after the defaulter-pays resources, and subsequently, in a second layer, once the prefunded DF assets of surviving members are depleted. That is, the second layer of SITG can be used before surviving members’ unfunded DF contributions.[4] The first and second layers of SITG are denoted by S and $\tilde{S}$ in this paper (Section 2).

We also consider so-called “monolayer” default waterfalls, whereby the IM pool is used for loss mutualization without any additional DF layer (Section 6). Some systemically important securities CCPs in the United States operate under this structure. Kuong and Maurin (2023, Proposition 3 and Corollary 1) show that CCP default waterfalls in their most general and abstract forms may be optimal when the collat-

[3] CCPs have failed in the past. The most well-known cases are the failures of the Caisse de Liquidation in Paris in 1974, the Kuala Lumpur Commodity Clearing House in 1983 and the Hong Kong Futures Guarantee Corporation in 1987. Tucker (2014) highlights the impact of the failure of the Hong Kong futures clearing house: “Basically, Hong Kong’s securities markets all stopped, affecting households and firms well beyond the community who had positions in stock-index futures.”

[4] When losses cannot be covered by prefunded financial resources, CCPs can often ask the surviving members to make additional contributions to the DF. These are referred to as “unfunded DF contributions”.

eral cost is not too high. Ghamami *et al* (2021) and Ghamami (2020) show that collateral in the form of IM may increase contagion and financial stability risks. Choi *et al* (2021) discuss the status of collateral rehypothecation and its impact on the global financial system.

As shown in the CCP surveys by Thiruchelvam (2022) and Walker (2023):

- SITG is often a very small fraction of member prefunded resources (for instance, SITG represents 1% of the DF at the United Kingdom's largest CCP for interest rate swaps, the London Clearing House (LCH));
- SITG levels vary widely across CCPs; and
- policy makers do not have a quantitative methodology for evaluating the sufficiency of SITG levels (Murphy 2017).

The goal of the present study is to address these shortcomings by introducing a sound, transparent and well-founded quantitative framework for determining the adequate size for a CCP's SITG; that is, the level of capital contribution to its default waterfall. Unlike bank regulation, CCP regulation is mostly principles-based (Ghamami 2015), and capital regulation of CCPs may not correspond to their risk profiles. Our proposed formulations of SITG can be viewed as risk-based lower bounds on the minimum CCP capital requirements (Section 5).

Conflicts of interest and agency problems are ubiquitous in OTC markets. They arise in different forms in centrally cleared markets. When left unmitigated, problems linked to CCP risk management may have adverse consequences on the CCP's financial stability. CCPs can be viewed as counterparty-credit-risk insurance providers. The classical moral hazard from this perspective is that clearing members may be incentivized to take on increasing amounts of counterparty credit risk. A well-designed loss mutualization scheme and adequate collateral requirements could mitigate this moral hazard. In the mechanism design approach of Biais *et al* (2016) and Bolton and Oehmke (2015), adequate levels of collateral (margin) can mitigate moral hazards of this type in derivatives markets.

Given default losses can be mutualized among surviving members, CCPs without sufficient SITG may not be incentivized to properly monitor counterparty credit risk. CCP risk management practices could subsequently become questionable. Well-designed SITG can mitigate this variation on the moral hazard. This agency problem can become subtle at member-owned CCPs. Unlike investor-owned CCPs, those under the ownership structure of a members' cooperative might be expected to naturally incentivize managers to put in place a robust risk management framework. This need not be the case, as we argue by drawing on the work of Hart and Moore (1996) and Hansmann (2013). A member-owned CCP could face collective decision-making complications that may ultimately lead to insufficient levels of SITG. We show that this problem can be exacerbated under CCPs with heterogeneous membership (Sections 2 and 3). Membership is not homogeneous at large clearinghouses. Our results can be contrasted with the contract-theoretic work of Huang (2019), in which a member-owned CCP is modeled as a welfare-maximizing social planner (ie, a public utility).

Taking the default waterfall as given, we develop an economic framework to analyze central clearing risk management agency problems and design SITG to mitigate them. We show that, conditional on a member's default, when $S = 0$, surviving members' DF assets are more exposed to losses compared with the losses the CCP could face under member prefunded resources. Quantifying the corresponding loss probabilities, we introduce incentive compatibility constraints (ICCs) and formulate SITG to mitigate the risk management moral hazards. In our setting, S, in its simplest form, is formulated as a percentage of the total DF (denoted by D in this paper).

Consider the CCP's tail exposure to each member conditional on that member's default. Ordering these (tail) loss exposure estimates, we assume for simplicity that member 1 creates the largest exposure.[5] We call the ratio of the CCP's largest exposure to its aggregate exposures the concentration ratio and denote it by c_1. We show that, in its simplest form, when

$$S = (1 - c_1)D,$$

some of the ICCs are satisfied, and the risk management incentives of the CCP and its members can become more aligned (Section 3.2).

We also show that when $\tilde{S} = 0$, members' unfunded DF contributions are more exposed to losses compared with the CCP loss exposures. This can distort risk management incentives. The moral hazard can be mitigated by formulating an $\tilde{S}$ that satisfies a set of incentive compatibility conditions (Section 3.3). As with our formulation of S, we show that $\tilde{S}$ is formulated as a percentage of D. We propose a quantitative approach for calibrating SITG (Section 4). In our framework the total SITG, $S + \tilde{S}$, can be expressed as a fraction of the total DF size. Our numerical studies in Section 5.1 indicate that for realistic parameters, this leads to SITG levels above 15%–20% of the total DF size. This in turn leads to estimates of a lower bound for CCP equity capital in terms of the total DF size.

The largest securities clearinghouses in the United States operate under the monolayer default waterfall. We show that monolayer CCPs may need to hold significantly higher levels of SITG to mitigate risk management agency problems (Section 6). We can approximate the ratio of the monolayer CCP SITG to the multilayer CCP SITG under similar ICCs. This ratio can be considered roughly equal to the ratio of

[5] We often refer to this member (member 1) as the largest member, or the member to which the CCP has the largest exposure.

total IM to the total DF under the multilayer default waterfall. In practice, the IM is usually at least 10 times the DF (Ghamami and Glasserman 2017). According to Walker (2023) and Thiruchelvam (2022), monolayer CCPs' capital contribution to the default waterfall is less than 1% of member prefunded resources. Our framework indicates that higher levels of SITG may be required to mitigate potential risk management incentive distortions.

Our findings also have implications for the adequacy of the bank capital requirements for exposure to CCPs developed by the Basel Committee on Banking Supervision (2023). Appendix A6 (online) shows that these CCP risk capital rules could be improved, as central clearing risks may be underestimated in the current regulatory regime.

The remainder of the paper is organized as follows. Section 2 reviews the typical multilayer default waterfall. It shows that CCPs may not be incentivized to allocate their own capital to the default waterfall. Section 3 develops our basic framework, which captures the risk management agency problems discussed in the previous section and shows they can be mitigated by holding adequate levels of SITG. Section 4 models the tail of the loss distributions with the Pareto distribution and develops a robust framework for formulating S and $\tilde{S}$. Section 5 introduces a lower bound for CCP regulatory capital. Section 6 analyzes the monolayer default waterfall and compares it with the multilayer waterfall. Section 7 states our concluding remarks and discusses the implications of our investigation.

2 DEFAULT WATERFALL AND CENTRAL COUNTERPARTY CAPITAL

After providing a brief overview of the default waterfall, we argue that, in the absence of government regulation, CCPs may not be incentivized to make adequate equity capital contributions to the default waterfall. We also note that SITG could be viewed as the minimum CCP regulatory capital requirement.

2.1 Default waterfall

Consider a CCP that clears transactions in an asset class for N clearing members indexed by $i = 1, \dots, N$. We denote by U_i the exposure of the CCP to member i over a given risk horizon, often referred to as the margin period of risk (MPOR). The exposure U_i is a positive random variable that in part captures member i's portfolio-value changes over the MPOR.

Each member i with open positions contributes an IM M_i to the CCP. At multilayer CCPs, the IM posted by each member may be used only to absorb losses arising from the member's portfolio, and it cannot be used to offset other members' losses or other losses incurred by the CCP. We discuss CCPs under the monolayer waterfall structure in Section 6. Regulatory guidelines require IM to cover the exposure with a certain confidence level, typically with a minimum of 99% (see Committee on Payment and Settlement Systems–Technical Committee of the International Organization of Securities Commissions 2012, Principle 6). We represent M_i as a quantile of U_i for some confidence level $1-q$, where $q \leqslant 0.01$.

The (residual) exposure net IM to member i is thus given by $(U_i - M_i)^+ = \max(U_i - M_i, 0)$. The magnitude of the CCP's exposure to member i's net IM in extreme but plausible scenarios is often modeled using a risk measure ρ associated with the random variable $(U_i - M_i)^+$ at confidence level $1-q_D$:

$$E_i = \rho_{q_D}((U_i - M_i)^+), \tag{2.1}$$

with $q_D < q$. Note that ρ can be the value-at-risk (VaR), expected shortfall, range VaR or any other loss-based risk measure (Cont *et al* 2013). In this paper, unless mentioned otherwise we use VaR:[6]

$$\rho_{q_D}((U_i - M_i)^+) = \mathrm{VaR}_{q_D}((U_i - M_i)^+).$$

Each member also contributes to the CCP's prefunded DF. We denote the contribution of member i to the DF by D_i, and the size of the total DF by D, where

$$D = \sum_{i=1}^{N} D_i.$$

Regulatory guidelines require that the DF covers potential losses incurred due to a given number of member defaults: at least one, and often two for systemically important CCPs (see Committee on Payment and Settlement Systems–Technical Committee of the International Organization of Securities Commissions 2012, Principle 4).[7] Denoting by $E^{(i)}$ the ith-largest exposure, we have

$$E^{(1)} = \max(E_i,\ i = 1, \dots, N) \geqslant E^{(2)} \geqslant \cdots \geqslant E^{(N)} = \min(E_i,\ i = 1, \dots, N).$$

The "Cover 1-based" DF leads to a prefunded DF given by the size of the CCP's largest tail exposure:

$$D = \max(E_i,\ i = 1, \dots, N) = E^{(1)}. \tag{2.2}$$

The Cover 2-based DF is intended to cover the simultaneous default of the two members that would jointly create the CCP's largest (tail) exposure. The Cover 2 DF can

[6] This is to simplify the exposition and focus on the main results. It is not difficult to carry out the analysis when ρ is taken as the expected shortfall.
[7] Systemically important securities CCPs in the United States operate under the Cover 1 DF rule.

be formulated as[8]

$$D = E^{(1)} + E^{(2)}. \tag{2.3}$$

To simplify the exposition, the analysis in the body of the paper mostly focuses on the Cover 1 DF. Appendix A5 (online) extend our analysis and results to the more general setting (ie, a Cover n DF; $2 \leqslant n \leqslant N$). Unlike IM, which became standardized to some extent at derivatives CCPs after the GFC, the modeling and sizing of the DF and its allocation to members vary considerably across CCPs (Cont 2015; Ghamami 2015; Ghamami and Glasserman 2017). Some derivatives CCPs allocate the DF to members proportionally to the tail exposures:

$$D_i = D \frac{E_i}{\sum_{j=1}^{N} E_j}. \tag{2.4}$$

Intuitively, this seems plausible, as the overall size of the DF depends on the magnitude of these exposures. However, other allocation schemes also exist. For instance, some CCPs allocate the total DF proportional to IM, trading volume, open interest or a weighted mixture of all these quantities.

The order in which the default waterfall's financial resources are used to absorb losses when a member defaults can be summarized as follows.

(1) The first layer of protection against losses is provided by the IM posted by the defaulting member.

(2) If the loss exceeds the IM contribution of the defaulting member, its prefunded DF contribution is used to cover any additional losses.

(3) If the loss exceeds the sum of the defaulting member's IM and DF contribution, the CCP makes a (capped) contribution to offset the remaining loss. This contribution is often referred to as the SITG. We denote the size of this first layer of SITG by S.

(4) The DF contributions of surviving members are used to absorb the potential remaining losses. These losses can be mutualized and allocated across members proportional to their contribution D_i to the DF.

[8] It can also be formulated as $\max\{\rho_{q_D}((U_i - M_i)^+ + (U_j - M_j)^+);\ i \neq j,\ i, j = 1, \dots, N\}$. As long as the subadditivity property holds, a Cover 2 DF formulated as (2.3) leads to a more conservative DF.

(5) Once the prefunded DF is exhausted, the CCP may use various recovery mechanisms to restore its funding resources (Cont 2015; Duffie 2015). These typically include

- an additional capital contribution by the CCP (we denote the size of this second layer of SITG by $\tilde{S}$);
- additional DF contributions by surviving members (known as assessments), capped at the level of their prefunded DF contribution; and
- other recovery measures, such as variation margin haircuts.

2.2 CCP capital contribution to the default waterfall

In the absence of SITG regulation, investor-owned CCPs may not be incentivized to make capital contributions to the default waterfall.[9] This can be shown in different ways.

In what follows we first illustrate the waterfall in a very simple and stylized way. Conditional on the default of member j, the CCP's loss up to this stage of the default waterfall can be written as

$$L = \min\{(U_j - M_j - D_j)^+, S\} + \min\{(U_j - M_j - D - S)^+, \tilde{S}\}. \qquad (2.5)$$

CCP revenue is proportional to the volume of cleared transactions. Consider an investor-owned CCP. Let V and ϕ denote the CCP's average clearing volume over a given period of time and the clearing fee. The CCP's expected net profit could then be approximated by

$$\phi V - E[L]. \qquad (2.6)$$

Suppose the CCP maximizes its expected net profits by choosing optimal levels of S and $\tilde{S}$. In the absence of capital constraints, the CCP solves this problem by setting $S = 0$ and $\tilde{S} = 0$. It is clear from the above formulation that the CCP chooses zero capital contribution.

If regulators require clearinghouses to contribute the minimum regulatory capital to the default waterfall, CCPs may then be incentivized to adjust their capital structure and improve their risk management frameworks so as to maximize (2.6) given the minimum regulator-enforced $S > 0$ and $\tilde{S} > 0$. For instance, suppose V is a decreasing function of the IM. That is, all else being equal, margin levels

[9] We consider two broad classes of ownership structure: outside ownership (which is the most common); and members' cooperatives. These are referred to as investor- and member-owned (or user-owned) CCPs, respectively. It can be illuminating to view these as capital and consumer cooperatives (Hansmann 2013).

above a regulatory minimum reduce the volume of trades the CCP can attract. Then, given regulator-driven $S > 0$ and $\tilde{S} > 0$, the profit maximization problem would be solved by, for example, choosing optimal levels of IM, S and $\tilde{S}$ above the regulatory minimum.

2.2.1 CCP objective function

The approximate net profit formulation (2.6) abstracts away the total level of CCP capital, costs associated with it and the cost of a CCP failure. We now sketch the augmented objective function of an investor-owned CCP in the presence of these costs.

Consider the case of a single representative CCP. Given the typical multilayer default waterfall, suppose that $E_{\mathrm{t}} = S + \tilde{S} + E_{\mathrm{s}}$ is the capital of the investor-owned CCP, where E_{s} is the portion of the CCP capital not allocated to the default waterfall. We also assume the unfunded DF is capped by a multiple of the prefunded DF, βD, with $\beta > 0$. Conditional on the default of member j, consider the loss to the CCP in excess of member resources and the CCP's total capital:

$$L_{\mathrm{e}} = (U_j - M_j - D - \beta(D - D_j) - E_{\mathrm{t}})^{+}. \tag{2.7}$$

In the presence of $E_{\mathrm{t}} > 0$ and conditional on the default of member j, the private profit-seeking objective of an investor-owned CCP would be to maximize

$$\phi V - E[L] - E[L_{\mathrm{e}}] - c(E_{\mathrm{t}}) - c_{\mathrm{p}} Q(S, \tilde{S}, E_{\mathrm{s}}), \tag{2.8}$$

where the first two terms come from (2.6); $c(E_{\mathrm{t}})$ is the social cost of CCP capital, with $c(\cdot)$ being an increasing convex function; c_{p} is the private cost of a CCP failure; and $Q(S, \tilde{S}, E_{\mathrm{s}})$ is the probability of such a failure, with $Q(\cdot)$ being a multivariate decreasing convex function. A basic and standard assumption in this paper is that CCP failures are costly for society. We assume that in the absence of capital requirements, CCPs do not fully internalize the costs of their own failures. That is, the social cost of a CCP failure, c_{s}, is larger than the private cost c_{p}. A second basic and standard assumption is that there is a social cost associated with having more CCP capital, and that the capital structure irrelevance principle (Modigliani and Miller 1958) fails for CCPs.[10]

[10] Greenwood *et al* (2017) use similar assumptions to formulate the cost of capital and bank failure in their setting. As in their study, the only cost of equity that is incorporated in the objective function (2.8) is the one associated with the stock of equity on the balance sheet. Under the simplifying assumption that the private costs of equity finance equal the social costs, Greenwood *et al* show that risk-based bank capital regulation can be optimal (in a steady state). We do not need to deviate from this simplifying assumption to show that, in the absence of SITG regulation, CCP equity capital levels can be socially suboptimal.

Now, suppose that E_t is given (eg, set by regulators) but its allocation to S, $\tilde{S}$ and E_s is left to the CCP. Then, it is not difficult to see that the CCP maximizes its objective by setting $S = \tilde{S} = 0$. That is, it does not allocate its own equity capital to the default waterfall. While the augmented objective function (2.8) is more comprehensive than the stylized net profit formulation (2.6), to see that investor-owned CCPs may not be incentivized to have any SITG, it suffices to focus on the simpler formulation (2.6). In the absence of SITG regulation, policy makers may underestimate the probability of CCP failure and so the corresponding social costs of such a failure. In what follows we show that, when $S = \tilde{S} = 0$, CCP risk management incentives can be distorted. This suggests that adequate levels of SITG can improve CCP risk management and subsequently decrease the CCP's default probability. The following example may be illuminating.

Suppose the true default probability of the CCP is represented by the logistic distribution function

$$Q(S, \tilde{S}, E_\mathrm{s}) = \frac{\exp(\zeta_0 + \zeta_1 S + \zeta_2 \tilde{S} + \zeta_3 E_\mathrm{s})}{1 + \exp(\zeta_0 + \zeta_1 S + \zeta_2 \tilde{S} + \zeta_3 E_\mathrm{s})}, \tag{2.9}$$

where $\zeta_i < 0$, $i = 0, 1, 2, 3$. In the absence of SITG regulation, the regulator (ie, the social planner) may obtain suboptimal levels of CCP equity capital by maximizing the following objective function:

$$\phi V - E[L] - E[L_\mathrm{e}] - c(E_\mathrm{t}) - c_\mathrm{s} \hat{Q}(E_\mathrm{t}), \tag{2.10}$$

where, instead of the CCP's true default probability function (2.9), the following inaccurate estimate of it is used:

$$\hat{Q}(E_\mathrm{t}) = \frac{\exp(\eta_0 + \eta_1 E_\mathrm{t})}{1 + \exp(\eta_0 + \eta_1 E_\mathrm{t})}, \tag{2.11}$$

with $\eta_0, \eta_1 < 0$.

2.2.2 Outside ownership versus members' cooperatives

It is well known that some clearinghouses are member-owned (or user-owned), such as the Options Clearing Corporation and subsidiaries of the Depository Trust and Clearing Corporation in the United States, and the Japanese Securities Clearing Corporation. Member-owned CCPs are often treated as public utilities or welfare-maximizing social planners in the existing contract-theoretic or mechanism design models of central clearing (see, for example, Huang 2019, Section 5).[11] Since member-owned CCPs are owned not by individuals but by other profit-seeking

[11] More generally, CCPs have sometimes been modeled as public utilities (see, for example, Biais *et al* 2016, p. 1677).

firms, economic analysis of this class of CCPs under the assumption they are welfare-maximizing public utilities may not have fruitful policy applications.

While the governance and ownership structures of CCPs are beyond the scope of this paper, we draw on the important work of Hart and Moore (1996) and Hansmann (2013). Their analyses of cooperatives, using different approaches, illustrate the importance of a "control-based" view of ownership and note that effective governance of a members' cooperative could be challenging and the cost of collective decision-making could be rather high under this ownership structure. They show that outside ownership can be more efficient than a members' cooperative when the membership becomes less homogeneous. Membership is not homogeneous by any measure at systemically important CCPs.

Member-owned CCPs in advanced economies are large businesses run by managers. Exerting effective control over management can be particularly difficult at a members' cooperative. We argue that SITG should be regulated at member-owned CCPs. Otherwise, risk management agency problems may adversely impact financial stability. In the absence of effective capital regulation, member-owned CCPs may allocate insufficient levels of SITG to the default waterfall due to collective decision-making complications and membership heterogeneity, and the fact that CCP equity capital could be more costly than collateral in the form of member DF contributions. In Section 3 we sketch the objective function of member-owned CCPs in comparison with (2.6) and illustrate that, unlike investor-owned CCPs, member-owned CCPs may have the incentive to allocate some level of SITG to the default waterfall. We also show how membership heterogeneity may lead to insufficient SITG levels.

2.2.3 CCP capital regulation

Principles 2, 4 and 15 of the Principles for Financial Market Infrastructures outline minimum regulatory capital requirements for CCPs and indicate that parts of CCPs' own financial resources should be allocated to the default waterfall (Committee on Payments and Market Infrastructures–Board of the International Organization of Securities Commissions 2017; Committee on Payment and Settlement Systems–Technical Committee of the International Organization of Securities Commissions 2012). Regulators and CCPs are then expected to specify the form and size of total regulatory capital and the amount that should be allocated to the default waterfall. In Europe, for instance, CCP capital requirements are the sum of four components: capital requirements for winding-down or restructuring activities; capital requirements for operational and legal risk; capital requirements for credit, counterparty and market risk;[12] and capital requirements for business risk (European Union 2013a). The

[12] This component of CCP capital requirements in Europe is formulated using Basel Committee on Banking Supervision rules for credit, counterparty and market risk capital requirements.

European Market Infrastructure Regulation (EMIR) then requires CCPs to allocate 25% of the total regulatory capital to the default waterfall (European Union 2013b; McLaughlin 2018). Under our proposed framework, $S + \tilde{S}$ can be viewed as a lower bound for CCP regulatory capital. It can also be directly compared with EMIR's 25% rule.

3 DEFAULT LOSSES AND SKIN IN THE GAME

In this section we compare default losses from the perspectives of the CCP and members and illustrate that the CCP and members are disproportionately exposed to default losses. The underlying economic argument is that when SITG is designed to lower the potential DF asset losses to members, risk management incentive distortions can be mitigated.

3.1 Member perspective

Consider the exposure of a surviving member to the default of another member. If member j defaults, the potential loss of DF assets of a nondefaulting member $i \neq j$ is given by

$$\underbrace{(U_j - M_j - D_j - S)^+}_{\text{loss allocated to the remaining DF}} \times \underbrace{\left(\frac{D_i}{D - D_j}\right)}_{\text{fraction allocated to member } i}. \tag{3.1}$$

If we limit member i's DF losses to its prefunded DF contributions, D_i, the resulting exposure of member i due to the default of member j becomes

$$L_i^j = D_i \min\left(\frac{(U_j - M_j - D_j - S)^+}{D - D_j}, 1\right). \tag{3.2}$$

Note that the ratio L_i^j / D_i (ie, the relative loss of DF assets) is the same for all nondefaulting members and depends only on the severity of the default event and on the DF allocation rule. Surviving members incur losses if the defaulting member's loss exceeds its IM, its DF and the first layer of SITG. That is,

$$L_i^j > 0 \iff U_j > M_j + D_j + S. \tag{3.3}$$

This result holds regardless of the rules used for sizing the DF and for its allocation across members.

We now consider the case of DF contributions being allocated proportional to tail exposures net IM, as in (2.4). Given (3.3), nondefaulting members will incur losses if

$$U_j > M_j + D \frac{E_j}{\sum_{k=1}^{N} E_k} + S.$$

In the Cover 1 case, where $D = E^{(1)}$, this inequality becomes

$$U_j > M_j + E_j \frac{E^{(1)}}{\sum_{k=1}^{N} E_k} + S, \tag{3.4}$$

where E_j is defined in (2.1). The right-hand side of (3.4) involves what we refer to as the concentration ratio,

$$c_1 = \frac{E^{(1)}}{\sum_{k=1}^{N} E_k}, \tag{3.5}$$

which measures the relative magnitude of the CCP's largest exposure. As will be further discussed below, this ratio can play an important role in the risk analysis of the default waterfall. Since $c_1 < 1$, in the absence of any CCP capital contribution the probability that nondefaulting members take a loss is always larger than q_D:

$$P\left(U_j > M_j + D \frac{E_j}{\sum_{k=1}^{N} E_k}\right) \geqslant P(U_j > M_j + E_j) = q_D.$$

Note that this is the case regardless of the magnitude of the default. In short, setting $S = 0$ gives

$$P(L_i^j > 0) \geqslant q_D, \tag{3.6}$$

where $i \neq j$.[13]

3.2 Skin in the game: first layer

Conditional on the default of member j, when $S = 0$ the potential loss to the CCP in the presence of IM and a prefunded DF is

$$L_0^j = (U_j - M_j - D)^+.$$

We can write[14]

$$P(L_0^j > 0) \leqslant q_D. \tag{3.7}$$

To simplify the notation, we assume hereafter that member 1 (N) is the member to which the CCP has the largest (smallest) exposure; that is, $E^{(1)} = E_1$ ($E^{(N)} = E_N$). Under the Cover 1 rule, we have

$$P(L_0^1 > 0) = q_D.$$

[13] If quantified, measured and monitored appropriately, loss probabilities associated with members' DF contributions can be used to construct credit ratings for CCPs' DFs.

[14] We use a subscript 0 in L_0 to represent losses from the perspective of the CCP.

We note that in the absence of any CCP capital contributions, nondefaulting members are more likely than the CCP to incur default losses:

$$P(L_i^j > 0) \geqslant q_D \geqslant P(L_0^j > 0). \tag{3.8}$$

This inequality captures an important conflict of interest between the CCP and members from a risk management perspective: in the absence of SITG, (nondefaulting) members are more exposed to default losses than the CCP. This moral hazard could be mitigated by reducing the loss probabilities associated with the prefunded DF assets of nondefaulting members. More specifically, we formulate S such that the following ICC is satisfied:

$$P(L_i^j > 0) \leqslant q_D. \tag{3.9}$$

To further elaborate on the incentive compatibility aspect of this constraint, consider two scenarios. In Scenario A we set

$$S_{\mathrm{L}} = (1 - c_1)D. \tag{3.10}$$

Let $c_N = E_N / (\sum_{i=1}^{N} E_i)$. In Scenario B we have

$$S_{\mathrm{U}} = (1 - c_N)D. \tag{3.11}$$

Note that $S_{\mathrm{L}} \leqslant S_{\mathrm{U}}$ as $c_N \leqslant c_1$.

SCENARIO A Conditional on the default of member 1, consider loss probabilities from the perspectives of member i and the CCP. Note that, given

$$P(L_i^1 > 0) = P(U_1 - M_1 > D_1 + S) \quad \text{and} \quad P(L_0^1 > 0) = P(U_1 - M_1 > D),$$

setting $S = D - D_1 = (1 - c_1)D$ gives

$$P(L_i^1 > 0) = P(L_0^1 > 0) = q_D. \tag{3.12}$$

That is, under (3.10) large counterparty default loss probabilities become perfectly aligned from the CCP and member perspectives. Moreover, we will show in Section 4 that under our framework, which is based on extreme value theory (EVT), the loss probability q_D in this scenario becomes an upper bound on member loss probabilities:[15]

$$P(L_i^j > 0) \leqslant q_D.$$

[15] In Section 4 we argue that the proposed EVT-based framework is the natural one to be considered for the design of SITG. It is intuitive economically and financially to model the tail of (conditional) loss distributions with the Pareto distribution.

While CCP and member risk incentives are fully aligned under the largest counterparty's default losses in Scenario A, nondefaulting members are more exposed to other (remaining) counterparties' default losses than the CCP:

$$P(L_i^j > 0) \geqslant P(L_0^j > 0), \quad j \neq 1, i. \tag{3.13}$$

We will return to this inequality shortly.

SCENARIO B When $S_{\mathrm{U}} = (1 - c_N)D$, the following inequality holds:

$$P(L_i^j > 0) = P(U_j - M_j > D + D_j - D_N) < q_D, \tag{3.14}$$

as $D_j \geqslant D_N$. That is, the overarching ICC (3.9) is satisfied. Moreover, under (3.11), we can write

$$P(L_i^j > 0) \leqslant P(L_0^j > 0) \tag{3.15}$$

for all $j \neq i$. That is, under S_{U} nondefaulting members are all less exposed to counterparty default losses than the CCP. In short, $[S_{\mathrm{L}}, S_{\mathrm{U}}]$ provides a range of values for the first layer of SITG whereby the moral hazard problem can be mitigated to different, quantifiable degrees. When policy makers aim to regulate the minimum equity capital, S_{L} could be a natural choice for SITG.

REMARK 3.1 That the CCP has low exposure to default losses,

$$P(L_0^j > 0) \leqslant q_D < q,$$

relies on the assumption that the DF has been sized adequately. These loss probabilities need not remain small if the DF does not capture client clearing risks properly. For instance, if in estimating the DF the CCP's exposure to member 1 conditional on its default, U_1, does not take into account the portfolios member 1 has cleared through the CCP on behalf of its customers, $P(L_0^1 > 0)$ could exceed q_D and q. Suppose member 1 defaults and its IM covers losses associated with its house (proprietary) account. Over the period of time until client accounts can be ported to a nondefaulting member, the CCP may need to make payments to member 1's customers. If the DF is not sized properly to cover losses that could arise due to member 1's default (or the default of some of its customers), the resilience of the CCP can be adversely impacted. This would all depend on the specifics of client clearing models, an important topic that is not addressed in this paper. The default waterfall should evolve proportionally to the risk profile of the CCP. Increased client clearing should increase IM, the DF and SITG adequately.

3.3 Skin in the game: second layer

The second layer of SITG could be viewed as a buffer against potential losses in members' unfunded DF assets. From the perspective of nondefaulting member i and conditional on the default of member j, the total loss in terms of member i's prefunded and unfunded DF assets can be represented by

$$\tilde{L}_i^j = L_i^j + (U_j - M_j - S - D - \tilde{S})^+ \frac{D_i}{D - D_j}. \tag{3.16}$$

When the unfunded DF assets are capped by D_i (or a multiple of D_i, denoted by βD_i; $\beta > 0$), the second term on the right-hand side above is replaced with the minimum between it and D_i (or βD_i). Given (3.16), the probability that potential losses to member i exceed its prefunded DF assets is given by

$$P(\tilde{L}_i^j > D_i) = P(U_j - M_j > D + S + \tilde{S}). \tag{3.17}$$

This is the likelihood that member i's unfunded DF resources would come into play due to the default of member j. This probability is bounded above by q_D when $j \neq 1$ or when S or $\tilde{S}$ is positive. Note that, when $S = \tilde{S} = 0$, the probability that the DF of member i is depleted due to the default of the largest member is equal to q_D:

$$P(\tilde{L}_i^1 > D_i) = q_D. \tag{3.18}$$

Given $S > 0$, we formulate $\tilde{S}$ to lower the likelihood that members' losses exceed their prefunded DF contributions. More specifically, consider a target loss probability (ie, an upper bound) associated with unfunded DF contributions, $\tilde{\pi}$, where $0 < \tilde{\pi} < q_D$. Given $S > 0$, we specify $\tilde{S}$ such that the following constraint is satisfied:

$$P(\tilde{L}_i^j > D_i) \leqslant \tilde{\pi}. \tag{3.19}$$

In short, under the most basic form of the incentive compatibility framework, S and $\tilde{S}$ can be formulated such that loss probabilities satisfy ICCs (3.9) and (3.19). We now demonstrate that (3.19) is grounded on economic arguments similar to those in the previous section. More specifically, taking the CCP's perspective, conditional on the default of member j, when $\tilde{S} = 0$ the potential loss to the CCP in excess of S and all prefunded and unfunded resources is

$$\tilde{L}_0^j = (U_j - M_j - D - S - \beta(D - D_j))^+. \tag{3.20}$$

Given $S > 0$, note that when $\tilde{S} = 0$ we have

$$P(\tilde{L}_i^j > D_i) > P(\tilde{L}_0^j > 0) \tag{3.21}$$

for any $j \neq i$. This inequality captures another important conflict of interest between the CCP and its members: in the absence of the second-layer SITG, members' potential losses in excess of their prefunded DF assets could be larger than the comparable potential loss to the CCP. The target loss probability $\tilde{\pi}$ will be chosen such that ICC (3.19) would mitigate this moral hazard. To elaborate on this second overarching and basic ICC, it will be illuminating to consider two scenarios. First, in Scenario A, we set

$$\tilde{S}_{\mathrm{L}} = \beta D(1 - c_1). \tag{3.22}$$

In Scenario B the second-layer SITG is formulated as follows:

$$\tilde{S}_{\mathrm{U}} = \beta D(1 - c_N). \tag{3.23}$$

Note that $\tilde{S}_{\mathrm{L}} \leqslant \tilde{S}_{\mathrm{U}}$.

Scenario A Suppose that $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$. Given $S > 0$, setting $\tilde{S}_{\mathrm{L}} = \beta D(1 - c_1)$ results in

$$P(\tilde{L}_i^1 > D_i) = P(\tilde{L}_0^1 > 0) = P(U_1 - M_1 > D + S + \beta(D - D_1)). \tag{3.24}$$

In other words, under (3.22) the CCP's and members' risk management incentives become fully aligned in terms of the potential largest-counterparty default losses that would exceed the prefunded resources and the first-layer SITG. As will be shown in Section 4, under our EVT (Pareto)-based framework,

$$P(\tilde{L}_i^j > D_i) \leqslant P(\tilde{L}_i^1 > D_i) \tag{3.25}$$

for $j \neq i \neq 1$. Consequently, under (3.22) the explicit ICC (3.24) along with the above EVT-driven inequality gives (3.19). However, we note that in Scenario A we have

$$P(\tilde{L}_i^j > D_i) \geqslant P(\tilde{L}_0^j > 0) \tag{3.26}$$

for any $j \neq i$. In other words, while $\tilde{S}_{\mathrm{L}} = \beta D(1 - c_1)$ mitigates the moral hazard associated with unfunded DF asset losses to some extent, members remain more exposed than the CCP to counterparty default losses that would exceed prefunded resources and the first-layer SITG. We will return to inequality (3.26) shortly.

Scenario B Suppose that $S > 0$ is given. Since $c_N \leqslant c_1$, setting $\tilde{S}_{\mathrm{U}} = \beta D(1 - c_N)$ gives

$$P(\tilde{L}_i^j > D_i) \leqslant P(\tilde{L}_0^j > 0) \tag{3.27}$$

for any $j \neq i$. It is useful to note that under (3.23) we have

$$P(\tilde{L}_i^N > D_i) = P(\tilde{L}_0^N > 0) = P(U_N - M_N > D + S + \beta(D - D_N)).$$

That is, under the more restrictive second-layer SITG (3.23), members become less likely than the CCP to incur counterparty default losses that would exceed their unfunded resources and the first-layer SITG. Here, the target loss probability upper bound associated with ICC (3.19) could continue to be viewed as $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$. That is, in this scenario both (3.19) and the more explicit and restrictive ICC (3.27) are satisfied. In sum, the second-layer SITG that belongs to the range $[\tilde{S}_{\mathrm{L}}, \tilde{S}_{\mathrm{U}}]$ can mitigate this variation of the moral hazard linked to the CCP's and members' tail risk management incentives to different quantifiable degrees. Given the regulatory focus on minimum equity capital requirements, policy makers could adopt and appropriately calibrate $\tilde{S}_{\mathrm{L}}$ as an economically sound choice for the second layer of SITG.

3.4 Member-owned CCPs

Exerting control on managers could be difficult at member-owned CCPs, in part due to collective-decision-making complications. This agency problem can be exacerbated under membership heterogeneity. Member-owned clearinghouses may have the incentive to allocate some levels of equity capital to the default waterfall. However, unregulated SITG levels may be insufficient and may in turn adversely impact risk management incentives.

Recall the expected net profit formulation (2.6) at investor-owned CCPs. We now introduce a variation of it that corresponds to member-owned CCPs. Suppose that $\psi_i V_i$ represents member i's gross profit from its trades in a volume of V_i that have been cleared through the CCP. For simplicity, we assume that $V = V_1 + V_2 + \cdots + V_N$.[16] Again for simplicity, suppose that all members receive an equal share of the CCP's profit. Then, conditional on the default of member j, member i's expected net profit can be written as

$$\frac{\phi V - E[L]}{N - 1} + (\psi_i V_i - E[\tilde{L}_i^j]),$$

where the second term inside the parentheses can be viewed as member i's "consumer surplus" in the sense of Hart and Moore (1996).[17] Note that $\tilde{L}_i^j$ is the total loss in terms of member i's prefunded and unfunded DF assets defined in (3.16). Taking

[16] Note that $\sum_{i=1}^{N} \psi_i$ need not be equal to ϕ as the fee structure at the CCP can be different from each member's profit-generating schemes from its trading activities.

[17] In any market, the total surplus can be viewed as the sum of the total producer surplus and the total consumer surplus (see Hart and Moore 1996, Section VII).

the typical multilayer default waterfall as given, member i maximizes expected net profit by choosing optimal levels of S and $\tilde{S}$:

$$\frac{\phi V}{N-1} + \psi_i V_i - \left(\frac{E[L]}{N-1} + E[\tilde{L}_i^j]\right). \tag{3.28}$$

Consequently, the expected net profit of the member-owned CCP conditional on the default of member j becomes

$$\left(\phi + \sum_{i \neq j} \psi_i\right) V - \left(E[L] + \sum_{i \neq j} E[\tilde{L}_i^j]\right). \tag{3.29}$$

These simple formulations highlight the basic fact that, unlike investor-owned CCPs, member-owned CCPs may not be incentivized to set $S = \tilde{S} = 0$.[18] To see this, consider expected losses in (3.28), and note that CCP managers' expected loss $E[L]$ can be viewed as an increasing function of S and $\tilde{S}$, while member i's expected loss $E[\tilde{L}_i^j]$ can be viewed as a decreasing function of S and $\tilde{S}$. So an optimal first- and second-layer SITG could be positive.[19]

Member and CCP expected net profit functions highlight the adverse impact of membership heterogeneity on SITG levels. To see this, consider the member expected net profit function (3.28). It then suffices to note that $\tilde{L}_i^j$ and thus $E[\tilde{L}_i^j]$ can be viewed as increasing functions of D_i. In other words, since larger members contribute more to the DF, their optimal levels of SITG can be larger than that of smaller members. Larger members would vote for higher levels of SITG, while smaller members would vote for lower SITG levels. In a heterogeneous member-owned CCP, reaching a consensus on an optimal level of SITG can be particularly challenging, as members with different levels of DF assets would vote for different levels of SITG.[20]

While conflicts of interest and agency problems in investor- and member-owned CCPs are not identical, the outcome could be similar: clearinghouses with socially suboptimal levels of SITG and CCP capital. This is particularly the case for systemically important CCPs that are also exposed to the too-big-to-fail problem.

We note that our SITG design framework directly applies to member-owned CCPs. For instance, consider inequality (3.8), which highlights the conflict of interest between the investor-owned CCP and its members from the risk management

[18] We can also easily see from (3.29) that modeling member-owned CCPs as welfare-maximizing social planners may not prove useful.

[19] The empirical results of Huang (2019) confirm that member-owned CCPs hold higher levels of SITG than investor-owned CCPs (see Figure 6 of Huang (2019), which uses 2015 quantitative disclosure data from CCPs).

[20] Note that control in the form of voting rights may be allocated according to a simple one-member-one-vote rule.

perspective. At the member-owned CCP, similarly, when $S = 0$, members are more exposed to default losses than the CCP, ie,

$$P(L_i^j > 0) \geqslant q_D \geqslant P(L_0^j > 0),$$

and this could disincentivize managers from adequately monitoring and managing concentrated risks at clearinghouses. Collective decisions-making problems and membership heterogeneity in the absence of governmental SITG regulation may lead to insufficient levels of SITG. Formulating S such that ICC (3.9) or (3.15) is satisfied could mitigate risk management incentive distortions and may also improve the collective decision-making process under this ownership structure.

REMARK 3.2 While the net profit formulations (3.28) and (3.29) abstract away the costs of CCP equity capital and CCP default, our results remain the same when these costs are incorporated into augmented objective functions. Recall the objective function of an investor-owned CCP (2.8). Given (3.28) under a members' cooperative ownership structure, the augmented objective function of member i conditional on the default of member j can be approximated by

$$\frac{\phi V}{N-1} + \psi_i V_i - \left(\frac{E[L] + E[L_{\mathrm{e}}] + c^{\mathrm{m}}(E_{\mathrm{t}}) + c_{\mathrm{p}}^{\mathrm{m}} Q^{\mathrm{m}}(S, \tilde{S}, E_{\mathrm{s}})}{N-1} + E[\tilde{L}_i^j]\right), \tag{3.30}$$

where L_{e} is defined in (2.7); $c^{\mathrm{m}}(E_{\mathrm{t}})$ is the social cost of the CCP capital under a members' cooperative ownership structure, with $c^{\mathrm{m}}(\cdot)$ being an increasing convex function; $c_{\mathrm{p}}^{\mathrm{m}}$ is the private cost of the CCP's default; and $Q^{\mathrm{m}}(S, \tilde{S}, E_{\mathrm{s}})$ is the CCP default probability, with $Q^{\mathrm{m}}(\cdot)$ being a multivariate decreasing convex function. Suppose that E_{t} is set by the regulators, and that, given E_{t}, member i maximizes (3.30) by allocating the total capital to S, $\tilde{S}$ and E_{s}. As discussed above, since $E[\tilde{L}_i^j]$ is a decreasing function of S and $\tilde{S}$, the optimal SITG level from the perspective of member i could be positive. However, SITG and total CCP capital levels may be socially suboptimal in the absence of governmental SITG regulation because, first, CCP and members do not fully internalize the cost of CCP failure, and second, under membership heterogeneity, different members could arrive at different (privately) optimal SITG levels, and this could lead to socially suboptimal SITG levels at the CCP. Our analysis suggests that SITG regulation may be required to effectively address risk management agency problems under this ownership structure.

4 A QUANTITATIVE FRAMEWORK FOR SKIN IN THE GAME

We use a flexible semiparametric approach to model the tail risk associated with default losses. This section introduces our framework in its most general form.

The proposed SITG formulations can mitigate risk management agency problems to different measurable degrees.

4.1 Modeling tail risk

Default losses often arise during extreme and distressed market conditions. To analyze the distribution of these losses, it is natural to model the tails of the loss distributions associated with CCP-member portfolios. A flexible and powerful semiparametric approach for modeling these distribution tails is to use the generalized Pareto distribution (GPD) (McNeil *et al* 2015, Chapter 7, Theorem 7.20). It is well known that the GPD becomes an ordinary Pareto distribution in the heavy tailed scenario, the case used in our study.

More specifically, we use the "threshold exceedances" approach to model the tail of the loss distribution (see Embrechts *et al* 1997; Tsay 2010).[21] We represent the conditional distribution of losses in excess of a high threshold as a Pareto distribution, whose tail exponent or shape parameter $\alpha > 1$ quantifies the heaviness of the tail. The natural threshold in our setting is IM. We thus assume that default exposures in excess of IM have a Pareto or power-law distribution. Specifically, the following assumption is used to derive our S and $\tilde{S}$ formulations.[22]

ASSUMPTION 4.1 (Pareto tail) *The CCP's exposure to member i conditional on its default, U_i, satisfies*

$$P(U_i - M_i > x \mid U_i \geqslant M_i) = \left(\frac{\kappa_i + x}{\kappa_i}\right)^{-\alpha} = 1 - F(x; \kappa_i, \alpha), \qquad (4.1)$$

where $M_i = \mathrm{VaR}_q(U_i)$*, and where*

$$F(x; \kappa, \alpha) = 1 - \left(\frac{\kappa + x}{\kappa}\right)^{-\alpha}$$

is the Pareto distribution with tail exponent (shape parameter) $\alpha > 1$ *and scale parameter* $\kappa > 0$.[23]

It is important to note that we are not assuming a parametric form for the entire loss distribution but only for tail events with a probability less than q (ie, for losses beyond IM). This assumption is consistent with risk models used by more advanced and sophisticated CCPs and is satisfied with high accuracy for many heavy-tailed distributions, such as a Student t, whose tails behave as (4.1) for high thresholds.

---

[21] The threshold exceedances approach is also referred to as peak-over-threshold (POT).

[22] Theorem 7.20 of McNeil *et al* (2015, p. 278) illustrates that "GPD is the canonical distribution for modeling excess losses over high thresholds".

[23] As discussed above, it is often the case that $q \leqslant 0.01$ in risk management applications.

Heavier tails correspond to lower values of the tail exponent α. Empirical studies indicate that this assumption is plausible for different asset classes, such as equity and credit portfolios, with α in the range 2–4 for equity (Cont 2001) and credit default swap (CDS) portfolios (Cont and Kan 2011). Higher values of the tail exponent correspond to equity and CDS indexes. Recall that

$$E_i = \text{VaR}_{q_D}((U_i - M_i)^+),$$

where $q_D < q \leqslant 0.01$. Assuming that DF is allocated according to E_i as in (2.4), we can write

$$P(U_i - M_i > E_i) = q_D = qP(U_i - M_i > E_i \mid U_i > M_i) = q\left(\frac{\kappa_i + E_i}{\kappa_i}\right)^{-\alpha},$$

where the last equality follows from Assumption 4.1. This gives

$$\kappa_i = \frac{\text{VaR}_{q_D}((U_i - M_i)^+)}{(q_D/q)^{-1/\alpha} - 1}. \tag{4.2}$$

This expression for κ_i shows that the scale parameter (κ_i) is proportional to the magnitude of losses in excess of IM. In what follows we assume that default loss distributions satisfy Assumption 4.1 with some tail exponent α and a scale parameter κ_i that may vary across members. Note that (4.2) and $E_i = \text{VaR}_{q_D}((U_i - M_i)^+)$ imply

$$\frac{E_i}{\sum_{j=1}^{N} E_j} = \frac{\kappa_i}{\sum_{j=1}^{N} \kappa_j} \tag{4.3}$$

for any $0 < q_D < 1$. We represent this ratio by c_i. We will return to concentration ratio c_1 shortly.

REMARK 4.2 Let σ_i^2 denote the variance of U_i. Following Lemma A2 in the online appendix, if we assume that $U_i/\sigma_i \sim T(0, \nu)$ has a mean-zero Student t distribution with $\nu > 1$ degrees of freedom, we will have

$$\frac{M_i}{\sum_{j=1}^{N} M_j} = \frac{E_i}{\sum_{j=1}^{N} E_j} = \frac{\kappa_i}{\sum_{j=1}^{N} \kappa_j}.$$

That is, when default exposure distributions over the MPOR are modeled by Student t distributions, the total DF can be allocated equivalently to members using E_i or M_i. The concentration ratio can also be approximated based on the IM. We will revisit this assumption in Section 6.

4.2 Skin in the game: first layer

We continue to assume that the CCP has its largest (smallest) exposure to member 1 (N). The following proposition gives the conditional probability distribution of the largest-counterparty default loss L_i^1 to member $i \neq 1$.

PROPOSITION 4.3 *Under Assumption 4.1 the probability distribution function of member i's loss conditional on the default of the member to which the CCP has the largest exposure is given by*

$$P(L_i^1 > x) = q\left[1 + \left(\left(\frac{q}{q_D}\right)^{1/\alpha} - 1\right)\left(c_1 + \frac{S}{D} + \frac{(1-c_1)}{c_i}\frac{x}{D}\right)\right]^{-\alpha}. \tag{4.4}$$

As illustrated above, in the absence of any distributional assumptions, when $S = 0$ we have $P(L_i^1 > 0) \geqslant q_D$ and $P(\tilde{L}_i^1 > D_i) = q_D$, which can be easily confirmed in the Pareto-based formulation. Working backward, we can compute the S that corresponds to a given target loss probability $\pi > 0$. For instance, suppose that $\pi = P(L_i^1 > 0)$. Solving for S yields that

$$P(L_i^1 > 0) = \pi \iff S = \left(\frac{(q/\pi)^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1} - c_1\right)D. \tag{4.5}$$

Our basic overarching objective in formulating S is to lower loss probabilities associated with member DF assets so as to achieve ICC (3.9). Setting $\pi \leqslant q_D$ satisfies this criterion, as will be shown shortly. Before doing so, we revisit Scenario A in the previous section, where aligning member and CCP largest-counterparty default loss probabilities is achieved by choosing S such that π becomes equal to $P(L_0^1 > 0) = q_D$. Choosing $\pi = q_D$ simplifies the above equation to (3.10),

$$S_{\mathrm{L}} = (1 - c_1)D,$$

which is the formulation we derived above without making any distributional assumptions. When S is sized according to (3.10), in the event of the default of the largest member, the probability that any surviving member incurs any losses will be q_D. In short, setting $S = (1 - c_1)D$ gives (3.12).

4.2.1 Concentration ratio and the DF

It is useful to highlight the nonlinear dependence of $S_{\mathrm{L}} = (1 - c_1)D$ on the DF and subsequent implications for the concentration ratio. Suppose that $q_D \in (0, q)$ is given. Viewing S_{L} as a function of $E_1 = \mathrm{VaR}_{q_D}((U_1 - M_1)^+)$, we have

$$S_{\mathrm{L}} = \left(1 - \frac{E_1}{\sum_{i=1}^N E_i}\right)E_1,$$

and it is straightforward to see that $E_1 = E/2$ (with $E = \sum_{i=1}^N E_i$) maximizes S_{L}. That is, a CCP with a concentration ratio of 0.5 would need to have the maximum level of $S_{\mathrm{L}} = E/4$ to satisfy ICCs (3.9) and (3.12). As the concentration ratio increases, the size of the DF increases, and so SITG will be sized based on lower percentages of the DF. Lower levels of c_1 lead to smaller DFs, and so SITG will be specified based on higher portions of the DF. For instance, $c_1 = 0.8$ gives $S_{\mathrm{L}} = 0.2D$, while $c_1 = 0.2$ gives $S_{\mathrm{L}} = 0.8D$. Now, suppose that c_1 is fixed and view S_{L} as a function of D. Note that, with a given c_1, the DF is a decreasing function of q_D. Lower q_D lead to larger DFs, and as D increases, the size of S_{L} will increase to ensure that member default loss probabilities are aligned with those of the CCP; that is, to ensure that ICCs (3.9) and (3.12) are satisfied.

4.3 Exposure to other defaults

Proposition 4.4 gives the probability distribution function of L_i^j, the loss to surviving member i in terms of DF assets conditional on the default of member $j \neq 1$.

PROPOSITION 4.4 *Under Assumption 4.1 the probability distribution function of member i's loss conditional on the default of j ≠ i is given by*

$$P(L_i^j > x) = q\left[1 + \left(\left(\frac{q}{q_D}\right)^{1/\alpha} - 1\right)\left(c_1 + \frac{S}{E_j} + \frac{x}{E_j}\frac{(1-c_j)}{c_i}\right)\right]^{-\alpha}. \qquad (4.6)$$

It follows from Propositions 4.3 and 4.4 that the highest level of tail risk corresponds to the default of member 1 with the highest stress loss over the margin:

$$P(L_i^j > x) \leqslant P(L_i^1 > x)$$

for any $j \neq i \neq 1$. That is, the largest DF exposure of members also corresponds to the largest exposure of the CCP. By contrast, if $S = 0$, then under Proposition 4.4 all members have the same loss probability regardless of the defaulter:

$$P(L_i^j > 0) = P(L_i^1 > 0) \geqslant q_D.$$

Using Propositions 4.3 and 4.4, it is not difficult to see that setting $S_{\mathrm{L}} = (1 - c_1)D$ results in the following bound for member loss probabilities:

$$P(L_0^j > 0) \leqslant P(L_i^j > 0) \leqslant q_D.$$

This shows that our formulation of S in Scenario A reduces the loss probabilities associated with member DF contributions and also aligns member and CCP large loss probabilities, in that (3.9) and (3.12) are guaranteed to hold.

Now, consider SITG in Scenario B. Recall that, under $S_{\mathrm{U}} = D(1 - c_N)$, ICCs (3.9) and (3.15) are satisfied:

$$P(L_i^j > 0) \leqslant P(L_0^j > 0) \leqslant q_D.$$

It is straightforward to use Proposition 4.4 to quantify the difference between loss probabilities under S_{L} and S_{U}.[24]

[24] In the context of CCP equity capital regulation, when regulatory SITG is viewed as the minimum amount of equity capital, policy makers could consider S_{L} instead of S_{U}.

4.4 Skin in the game: second layer

In the design and analysis of $\tilde{S}$ we use the following proposition, which gives our Pareto-based formulation of member and CCP loss probabilities ((3.17) and (3.21)) discussed in Section 3.[25]

PROPOSITION 4.5 *Under Assumption 4.1, and given $S, \tilde{S} > 0$, the probability that member i's loss conditional on the default of $j \neq i$ exceeds its prefunded DF resources is given by*

$$P(\tilde{L}_i^j > D_i) = q\bigg[1 + \bigg(\bigg(\frac{q}{q_D}\bigg)^{1/\alpha} - 1\bigg)\bigg(\frac{D}{E_j} + \frac{S}{E_j} + \frac{\tilde{S}}{E_j}\bigg)\bigg]^{-\alpha}. \tag{4.7}$$

Also, conditional on the default of member j and in the absence of $\tilde{S}$, the probability that the loss to the CCP exceeds (a given) $S > 0$ and the members' prefunded and unfunded DF assets is given by

$$P(\tilde{L}_0^j > 0) = q\bigg[1 + \bigg(\bigg(\frac{q}{q_D}\bigg)^{1/\alpha} - 1\bigg)\bigg(\frac{(1+\beta)D}{E_j} + \frac{S}{E_j} - \beta c_1\bigg)\bigg]^{-\alpha}. \tag{4.8}$$

Proposition 4.5 shows that the likelihood that the loss to the CCP exceeds S plus the members' prefunded and unfunded DF assets has a maximum conditional on the default of member 1, the member to which the CCP has the largest exposure. Similarly, the probability that the loss to a nondefaulting member exceeds its prefunded DF assets is at its highest conditional on the default of member 1.

Suppose that S is formulated according to (4.5). When $\tilde{S} = 0$, we can write $P(\tilde{L}_i^1 > D_i) \leqslant \pi \leqslant q_D$. We denote the upper bound for $P(\tilde{L}_i^1 > D_i)$ that corresponds to $\tilde{S} = 0$ and S formulated as in (4.5) by $\tilde{\pi}_0$. For instance, when $\pi = q_D$ (ie, when $S_{\mathrm{L}} = (1 - c_1)D$), setting $\tilde{S} = 0$ gives

$$\tilde{\pi}_0 = q\bigg[1 + \bigg(\bigg(\frac{q}{q_D}\bigg)^{1/\alpha} - 1\bigg)(2 - c_1)\bigg]^{-\alpha}. \tag{4.9}$$

This quantity could be interpreted as follows: if all members accept the risk of their DF assets being depleted with probability $\tilde{\pi}_0$, then the second-layer SITG will not be required.[26]

[25] The proof of Proposition 4.5 is omitted as the method of proof follows that for Propositions 4.3 and 4.4.

[26] We note that $\tilde{\pi}_0$ is an increasing function of c_1, which achieves its hypothetical minimum at $c_1 = 0$.

Now, working backward and given our formulation of S with the target loss probability $\pi \leqslant q_D$, we can specify the $\tilde{S}$ that corresponds to a given target loss probability $\tilde{\pi}$, where $\tilde{\pi} \leqslant \tilde{\pi}_0$. For instance, suppose that $\tilde{\pi} = P(\tilde{L}_i^1 > D_i)$. Solving for $\tilde{S}$ gives that

$$P(\tilde{L}_i^1 > D_i) = \tilde{\pi} \iff \tilde{S} = \left(\frac{(q/\tilde{\pi})^{1/\alpha} - (q/\pi)^{1/\alpha}}{(q/q_D)^{1/\alpha} - 1} + c_1 - 1\right)D, \qquad (4.10)$$

where $\tilde{\pi} < \tilde{\pi}_0 \leqslant q_D$.[27] Note that setting $\pi = q_D$ and so $S_{\text{L}} = (1 - c_1)D$ gives that

$$P(\tilde{L}_i^1 > D_i) = \tilde{\pi} \iff \tilde{S} = \left(\frac{(q/\tilde{\pi})^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1} + c_1 - 2\right)D. \qquad (4.11)$$

In our Pareto-based framework, designing S and $\tilde{S}$ with target loss probabilities π and $\tilde{\pi}$ ensures that the basic and overarching ICC (3.19) is satisfied:

$$P(\tilde{L}_i^j > D_i) \leqslant \underbrace{P(\tilde{L}_i^1 > D_i)}_{\tilde{\pi}} < q_D$$

for any $j \neq i \neq 1$. In sum, our formulation of S and $\tilde{S}$ in this section satisfies the ICCs (3.9), (3.12) and (3.19) developed above and will result in

$$P(\tilde{L}_i^j > D_i) < P(L_0^j > 0) \leqslant P(L_i^j > 0) \leqslant q_D. \qquad (4.12)$$

The last two inequalities follow from our results in Section 3, and the first inequality follows from the definitions of $\tilde{L}_i^j$ and L_0^j.

REMARK 4.6 It is important to note that under Assumption 4.1 $\tilde{S}_{\text{L}}$ and $\tilde{S}_{\text{U}}$, which were introduced in the previous section, can be viewed as special cases of our general Pareto-based framework. To see this, consider Scenario A in the previous section and note that (4.10) can also be written as

$$P(\tilde{L}_i^1 > D_i) = \tilde{\pi} \iff \tilde{S} = \left(\frac{(q/\tilde{\pi})^{1/\alpha} - (q/\pi)^{1/\alpha}}{(q/q_D)^{1/\alpha} - 1} - 1\right)D - S$$

for any $S > 0$. When the target loss probability is set to $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$, Proposition 4.5 and the above formulation result in $\tilde{S}_{\text{L}} = \beta D(1 - c_1)$. Now, consider Scenario B, where the specific incentive compatibility objective is satisfying (3.27). It is clear that $\tilde{S}_{\text{U}} = \beta D(1 - c_N)$ satisfies (3.27). It is also clear that under $\tilde{S}_{\text{U}}$ the basic ICC (3.19) holds with $P(\tilde{L}_i^1 > D_i) = \tilde{\pi}$.

[27] In the formulation of $\tilde{S}$ the first term on the right-hand side is positive because the target probability $\tilde{\pi}$ is set to be less than $\tilde{\pi}_0$.

5 CAPITAL REGULATION

Using the EVT (Pareto)-based framework, we introduce a lower bound for the CCP capital requirements and give a numerical example to show how this varies as a percentage of the DF. In Appendix A6 our framework is used to test the adequacy of Basel CCP risk capital rules.

5.1 Minimum CCP capital requirements

As discussed above, the CCP's equity capital contribution to the default waterfall could be viewed as a lower bound on its regulatory capital. Our framework gives the following lower bound for the minimum CCP capital requirements:

$$S + \tilde{S} = \left[\frac{(q/\tilde{\pi})^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1} - 1\right] D, \tag{5.1}$$

where S and $\tilde{S}$ are the first- and second-layer SITG given in (4.5) and (4.10). Also, recall that the first target loss probability $\pi = P(L_i^1 > 0)$ and the second target loss probability $\tilde{\pi} = P(\tilde{L}_i^1 > D_i)$ satisfy $\tilde{\pi} < \pi \leqslant q_D < q$. As discussed above, a simple way to ensure that $\tilde{S} > 0$ is by setting $\tilde{\pi} < \tilde{\pi}_0$, where $\tilde{\pi}_0$ is given in (4.9) and derived under $\tilde{S} = 0$ and $S = (1 - c_1)D$.

It is not difficult to show that, as the number of members increases, $S + \tilde{S}$ increases under our framework. To see this, note that $\tilde{\pi}_0$ is an increasing function of c_1. So, when the number of members increases, c_1 decreases; this would reduce $\tilde{\pi}_0$, which in turn reduces the target loss probability $\tilde{\pi}$ in our SITG formulation. And when $\tilde{\pi}$ decreases, the total level of $S + \tilde{S}$ increases. In other words, all else being equal, as the number of members increases, higher levels of SITG are required to mitigate risk management agency problems. This can be contrasted with the work of Kuong and Maurin (2023), where a major implication of their contract-theoretic model rationalizes the documented empirical observation that larger CCPs allocate smaller amounts of their own capital to the default waterfall.

It is worth emphasizing that our proposed CCP regulatory capital lower bound (5.1) is grounded on the incentive compatibility framework developed above. It also depends only on parameters and variables easily accessible to and monitored and controlled by CCPs and their regulators. The following example can offer insight, as it shows how the proposed lower bound varies as a function of $\tilde{\pi}, q_D, q$ and α.

EXAMPLE 5.1 Table 1 reports the ratio of the lower bound on regulatory capital to the DF for different values of α, q, q_D and $\tilde{\pi}$. First, note that D is governed by the confidence level $1 - q_D$ used to define DF stress scenarios. Note that D is a decreasing function of q_D. So, all else being equal, $(S + \tilde{S})/D$ can be viewed as an increasing function of q_D.

TABLE 1 The total SITG, $S+\tilde{S}$, as a fraction of DF D, computed from (5.1), for different values of parameters α, q, q_D and $\tilde{\pi}$.

$(S+\tilde{S})D$	α	q	q_D	$\tilde{\pi}$
0.67	2	100	50	35
0.18	2	100	50	45
0.09	2	200	100	95
1.62	2	100	30	10
2.51	2	100	80	50
17.32	2	100	80	10
3.44	3	100	50	10
0.61	3	100	50	35
1.34	3	100	30	10
0.36	4	100	50	40
0.59	4	100	50	35
0.67	4	80	60	50
3.17	4	70	40	10
0.3	4	100	80	75
0.16	5	100	50	45
0.57	5	100	50	35
2.93	6	100	50	10
0.57	6	100	50	35
0.62	6	100	80	70

q, q_D and $\tilde{\pi}$ are given in basis points.

Recall that to have $\tilde{S} > 0$, $\tilde{\pi} < \tilde{\pi}_0$ must hold. Table 1 reports the values of $\tilde{\pi}$ for which there exists $c_1 \in (0,1)$ and so $\tilde{\pi}_0 \in (0, q_D)$, where $\tilde{\pi} < \tilde{\pi}_0$ holds. For instance, consider the first row of the table, with $\alpha = 2$, $q = 0.01$ and $q_D = 0.005$. Setting $c_1 = 0.01$ gives $\tilde{\pi}_0 = 0.003$, while setting $c_1 = 0.9$ gives $\tilde{\pi}_0 = 0.0047$. So there exists a $\tilde{\pi}_0$ for which $\tilde{\pi} = 0.0035$ falls below $\tilde{\pi}_0$.

In interpreting Table 1, it is also useful to view α, q and q_D as given and note that higher target loss probabilities $\tilde{\pi}$ are generally associated with lower $(S+\tilde{S})/D$ ratios. In other words, to obtain lower loss probabilities, the lower bound on the regulatory capital $S+\tilde{S}$ should be formulated as a higher percentage of DF D. For instance, consider the two rows associated with $\alpha = 5$. To achieve $\tilde{\pi} = 0.0045$, the lower bound on capital requirements becomes $0.16D$, while lowering this loss probability by roughly 22% to $\tilde{\pi} = 0.0035$ requires a more than 250% increase in CCP capital contributions, $S+\tilde{S} = 0.57D$.

As discussed in Section 1, this numerical example shows that a total level of SITG below 15%–20% of the DF cannot be produced in the realistic part of our model parameter space. This contrasts with the current practice, which has been empirically summarized by Thiruchelvam (2022) and Walker (2023). The ratio of total IM to the total DF may also vary widely across CCPs: IM can be 10 times larger than the DF (Ghamami and Glasserman 2017). Using 2015–17 CCP quantitative disclosure data, Huang (2019) estimates an average SITG of about US\$38 million and an average IM of about US\$14 billion across 9–10 investor-owned CCPs. When the DF is 10% of IM, SITG becomes 2.7% of the DF. Under our framework, this level of SITG may not adequately mitigate CCP risk management incentive distortions.

The current regulatory regime is not risk-based; that is, it is not based on economic or financial-economics analysis. Our framework could be used to improve CCP capital regulation.

REMARK 5.2 The CCP regulatory capital lower bound in the more general Cover n DF setting is derived in Appendix A5. Systemically important derivatives CCPs often operate under a Cover 2 DF, for which the lower bound becomes

$$S_2 + \tilde{S}_2 = \left[\left(\frac{(q/\tilde{\pi}_2)^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1}\right)\left(\frac{c_1}{c_1 + c_2}\right) - 1\right] D_{s,2}. \tag{5.2}$$

This follows from (A.25) in the appendix. Recall that in the Cover 1 case we have $D = E_1$, and the total DF is $D_{s,2} = E_1 + E_2$ in the Cover 2 case.[28] It is important to note that under similar incentive compatibility structures, we have $\tilde{\pi}_2 < \tilde{\pi}$. This can result in $S_2 + \tilde{S}_2 \geqslant S + \tilde{S}$.

5.2 Optimal capital regulation

The lower bound on CCP equity capital (5.1) focuses on ICCs and central clearing risk management agency problems. It abstracts away the costs of CCP capital and CCP failure. Consider the case of a single representative CCP. Recall our sketch of the CCP objective function under outside ownership (see (2.8)). Social welfare can be represented by

$$\phi V - E[L] - E[L_{\mathrm{e}}] - c(E_{\mathrm{t}}) - c_{\mathrm{s}} Q(S(\pi), \tilde{S}(\tilde{\pi}), E_{\mathrm{s}}), \tag{5.3}$$

where, under the incentive compatibility framework, the first layer of SITG, $S(\pi)$, can be viewed as a decreasing function of the target loss probability π, and given π, the second layer of SITG, $\tilde{S}(\tilde{\pi})$, is a decreasing function of the second target loss probability $\tilde{\pi}$. Given members' IM and DF assets, the social planner's objective would then be to find the optimum SITG and E_{s} that maximize (5.3). In addition to IM and the DF, if the Pareto tail exponent, α, is also given, the social planner's

[28] As described in Appendix A5, in the more general Cover n DF setting, we have further simplified the notation by assuming that $E_N \leqslant E_{N-1} \leqslant \cdots \leqslant E_2 \leqslant E_1$. That is, under the Cover n rule, we write $D = \sum_{i=1}^{n} E_n$.

problem could be equivalently viewed as finding the optimum π, $\tilde{\pi}$ and E_s that maximize (5.3). Optimal SITG levels could then correspond to target loss probabilities under which some of the ICCs may be satisfied and some may not hold from the members' perspective. Optimal SITG levels are beyond the scope of this paper, which instead, focuses on developing an incentive compatibility framework that incorporates SITG into policy makers' revised objective function. That is, given the CCPs' default waterfall, we replace the social planner's inaccurate objective function (2.10) with a more accurate one (5.3) and link S and $\tilde{S}$ to a set of ICCs under which some of the CCP risk management agency problems can be mitigated.

6 MONOLAYER DEFAULT WATERFALL

Large derivatives CCPs do not often mutualize the pool of IM to cover defaulting member losses. It is the DF, the layer of collateral collected in addition to IM, that can be mutualized to cover losses. The default waterfall at some CCPs, in particular the largest securities clearinghouses in the United States, could be different. Securities CCPs in the United States often collect IM from their members and mutualize the pool of IM to cover losses. Total IM under the monolayer default waterfall plays the role of the DF at CCPs with the more typical multilayer waterfall.

In this section we analyze the monolayer default waterfall by modeling and estimating losses from the perspectives of a CCP and its members. We show, that compared with CCPs with the multilayer waterfall structure, clearinghouses with a monolayer default waterfall may need to have significantly higher levels of SITG to mitigate moral hazards linked to risk management incentives.

Monolayer CCPs have become increasingly important, as a broader central clearing mandate is a critical element of the government securities-market reform programs initiated by the G30's Working Group on Treasury Market Liquidity after the emergence of the Covid-19 pandemic.[29]

6.1 CCP perspective

In the absence of SITG, the exposure of a monolayer CCP conditional on the default of member j can be written as

$$\check{L}_0^j = (U_j - M)^+,$$

where $M = \sum_{i=1}^{N} M_i$ and $j = 1, \dots, N$. To compare monolayer and multilayer default waterfalls, we make the natural assumption that U_j, the random variables representing the CCP's exposure to member defaults, are drawn from the same

[29] See the original G30 report (Group of Thirty 2021) and the subsequent "status update" report (Group of Thirty 2022).

distribution under both waterfall structures. Recall that the CCP's exposure to the default of a member under the more typical multilayer waterfall structure is written as $L_0^j = (U_j - M_j - D)^+$. As will be explained shortly, in practice, it is often the case that

$$P(\check{L}_0^j > x) \leqslant P(L_0^j > x) \tag{6.1}$$

for any $x \geqslant 0$. In other words, the monolayer CCP is less exposed to the default of its members than the multilayer CCP. It is well known that, in CCPs operating under the multilayered default waterfall, total IM can be notably larger than the total DF. In fact, M can be more than 10 times larger than D, especially in large CCPs (Ghamami and Glasserman 2017).[30] Note that (6.1) holds when

$$M - M_j \geqslant D \tag{6.2}$$

for any $j = 1, \dots, N$. This condition is satisfied unless a CCP has an unrealistically high concentration ratio c_1. It should be intuitively clear that, since collateral in the form of IM often dominates DF, sharing IM to cover losses could be beneficial from the CCP's perspective.[31]

6.2 Member perspective

It is also straightforward to see that, compared with multilayer CCPs, in monolayer CCPs, surviving members are more exposed to the defaulting member losses. Note that SITG under the monolayer waterfall structure, denoted by $\check{S}$, comes into play right after the defaulters' IM. Conditional on the default of member j, potential losses in terms of the IM contribution of surviving member i can be written as

$$(U_j - M_j - \check{S})^+ \frac{M_i}{M - M_j},$$

where $i \neq j$. In the scenario where surviving member losses cannot exceed M_i, we have

$$\check{L}_i^j = M_i \min\left(\frac{(U_j - M_j - \check{S})^+}{M - M_j}, 1\right). \tag{6.3}$$

Comparing the two waterfall structures, we now show that members could incur larger losses when the IM pool is mutualized in the absence of an additional and separate prefunded DF.

---

[30] According to the Global Association of Central Counterparties (CCP12 2023), the total required IM for a selected 24 CCPs was US$956.6 billion, and the total required DF was US$49.1 billion, in 2022 Q4. Aggregated over 24 CCPs, the ratio of IM to the DF was around 19.5 at the end of 2022.

[31] Our analysis does not take into account recovery schemes such as variation margin haircuts (Cont 2015).

PROPOSITION 6.1 *If the SITG is sized similarly under both types of waterfall structures, we have*

$$P(\check{L}_i^j > 0) > P(L_i^j > 0), \tag{6.4}$$

where $i \neq j$, and $\check{L}_i^j$ and L_i^j are defined in (6.3) and (3.2), respectively. Let σ_i^2 denote the variance of U_i. Further, if $U_i/\sigma_i \sim T(0,\nu)$ has a mean-zero Student t distribution with $\nu > 1$ degrees of freedom, then for any loss level $x > 0$,

$$P(\check{L}_i^j > x) > P(L_i^j > x). \tag{6.5}$$

REMARK 6.2 Student t distributions belong to the class of elliptical distributions, which encompasses a large battery of models used in finance and economics, especially in risk management (Andersen and Dickinson 2018; Cont and Kan 2011; Ghamami and Glasserman 2019; Ivanov 2017). Student t distributions include the normal distribution as a limiting case (for $\nu \to \infty$). Since U_i, the CCP's exposure to member i, captures in part the portfolio-value changes over the MPOR, whose length is often up to five days, it is not unrealistic to assume that U_i has a mean of zero.

6.3 CCP capital contribution

If the CCP does not allocate its own capital to the default waterfall,[32] we have

$$P(\check{L}_i^j > 0) = P(U_j > M_j) = q,$$

where $1-q$ is the confidence level associated with the VaR-based IM. That is, conditional on any member's default, the probability that a surviving member's IM would incur losses is q. Comparing this member loss probability q with the CCP's loss probability conditional on a member's default,

$$P(\check{L}_0^j > 0) = P\biggl(U_j > M_j + \sum_{i \neq j} M_i\biggr),$$

it is clear that members could incur disproportionately larger losses than the CCP. To perfectly align largest-counterparty default loss probabilities between the CCP and its members, the CCP should contribute $\sum_{i=2}^{N} M_i$ to the loss waterfall. That is, setting $\check{S} = M - M_1$ gives

$$P(\check{L}_i^1 > 0) = P(\check{L}_0^1 > 0) \tag{6.6}$$

[32] Securities CCPs often have only one layer of SITG, which is denoted by $\check{S}$ in our analysis. It is not difficult to extend our analysis to formulate the second layer of SITG for monolayer CCPs.

for any $i \neq 1$. Note that condition (6.6) is analogous to ICC (3.12), which is satisfied under (3.10) in multilayer CCP. We will return to this formulation of SITG toward the end of this section.

We now use our Pareto-based framework to draw a useful comparison between monolayer and multilayer default waterfalls. We continue to assume that exposures U_i are drawn from similar distributions, and that E_i are quantified based on a given confidence level $1-q_D$ under both waterfall structures. Using Assumption 4.1 and employing the method of proof for Proposition 4.4, we have

$$P(\check{L}_i^j > 0) = q\left[1 + \left(\left(\frac{q}{q_D}\right)^{1/\alpha} - 1\right)\frac{\check{S}}{E_j}\right]^{-\alpha},$$

which implies that $P(\check{L}_i^j > 0) \leqslant P(\check{L}_i^1 > 0)$ for any $j \neq i \neq 1$. We now specify SITG corresponding to a given target loss probability $\check{\pi}$. For instance, suppose that $\check{\pi} = P(\check{L}_i^1 > 0)$, where $\check{\pi} \leqslant q_D$, associated with the largest member's default. Solving for $\check{S}$ yields

$$P(\check{L}_i^1 > 0) = \check{\pi} \iff \check{S} = \left(\frac{(q/\check{\pi})^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1}\right)E_1. \tag{6.7}$$

Choosing $\check{\pi} = q_D$ simplifies the above equation to

$$\check{S}_{q_D} = E_1. \tag{6.8}$$

That is, $\check{S}_{q_D} = D$ under the Cover 1 DF rule. Consequently, if SITG is formulated according to (6.8), then in the event of the default of the largest member, the probability that a surviving member would incur any losses will be $q_D < q$. This formulation of SITG is illuminating as it provides a way to directly compare monolayer and multilayer CCPs in terms of the required capital contribution to the waterfall that would guarantee similar default exposures from the perspective of members. That is, the two waterfall structures result in equal member loss probabilities (conditional on the largest member default),

$$P(\check{L}_i^1 > 0) = P(L_i^1 > 0) = q_D < q,$$

when the monolayer CCP operates under $\check{S}_{q_D} = D$ while the multilayer CCP contributes $S = (1 - c_1)D$. Given that $0 < c_1 < 1$, this result shows that policy makers may need to require higher levels of SITG at monolayer CCPs so that members are exposed to similar levels of default risk under both types of waterfall structures. All else being equal, the higher the concentration ratio, the greater the difference between CCP equity capital contributions under monolayer and multilayer default waterfalls.

The incentive compatibility framework can also be used more directly to compare monolayer and multilayer CCPs. Recall ICC (3.12) under the multilayer default waterfall in Scenario A. The analogous ICC for the monolayer CCP can be written as

$$P(\check{L}_i^1 > 0) = P(\check{L}_0^1 > 0) = \check{\pi}. \tag{6.9}$$

As discussed above, the above ICC is satisfied under $\check{S} = M - M_1$, which can be written as

$$\check{S}_{\mathrm{L}} = (1 - c_1)M, \tag{6.10}$$

by Lemma A2 in the online appendix, when we assume that $U_i/\sigma_i \sim T(0, \nu)$ has a mean-zero Student t distribution with $\nu > 1$ degrees of freedom. We note that under (6.10) the following basic and overarching ICC in the monolayer case is also satisfied:

$$P(\check{L}_i^j > 0) \leqslant P(\check{L}_0^1 > 0) \tag{6.11}$$

for any $j \neq i \neq 1$. It is important to note that the above economic arguments were initially used in the design of SITG that led to $S_{\mathrm{L}} = (1 - c_1)D$ under the multilayer waterfall. In short, when normalized exposures U_i have heavy (Pareto) tails, ICCs require

$$\check{S}_{\mathrm{L}} = \frac{M}{D} S_{\mathrm{L}}. \tag{6.12}$$

In other words, the capital contribution of the monolayer CCP to the default waterfall may need to be several multiples of that of a similar CCP with a multilayer waterfall.

To complete our comparative analysis of incentive structures associated with monolayer and multilayer CCPs, recall Scenario B under the more typical default waterfall, where $S_{\mathrm{U}} = (1 - c_N)D$ would guarantee that ICC (3.15) holds. We note that setting

$$\check{S}_{\mathrm{U}} = M - M_N = (1 - c_N)M \tag{6.13}$$

leads to a similar incentive structure for the monolayer CCP: under (6.13), the analogous ICC,

$$P(\check{L}_i^j > 0) \leqslant P(\check{L}_0^j > 0), \tag{6.14}$$

holds for any $j \neq i \neq 1$. Consequently, when enforcing this second incentive structure for monolayer and multilayer CCPs, we arrive at the same result, (6.12). That is, the SITG of the monolayer CCP would need to be M/D times larger than the SITG under the multilayer default waterfall.

In summary, our results illustrate that, compared with multilayer clearinghouses, CCPs operating under the monolayer structure should allocate more capital to the default waterfall.

7 CONCLUDING REMARKS

Recent distress in the banking sector highlights that post-GFC recovery and resolution frameworks may not work well in practice and may need to be improved. The Silicon Valley Bank (SVB) collapsed in March 2023. While the SVB was subject to bank-level resolution planning, and most of its assets were held in the bank, the resolution plans could not be implemented successfully (Clancy 2023). Effective CCP capital and SITG regulation is at least as important as improving CCP recovery and resolution frameworks.

We have proposed a robust and objective framework that can be used for designing CCP SITG requirements. Our framework is grounded in ICCs that capture central clearing risk management agency problems. The proposed SITG formulas are simple and readily implementable using data available to CCPs and regulators. Comparing our SITG formulations with CCP public data and the empirical evidence from recent CCP quantitative disclosures (Ghamami and Glasserman 2017; Huang 2019; Thiruchelvam 2022; Walker 2023), we conclude that investor- and member-owned CCPs may need to allocate more capital to default waterfalls.

Central clearing will play a key role in the reform of the US Treasury market. The resilience of clearinghouses that will be at the center of the US Treasury market is of critical importance.[33] To diversify the supply of Treasury market liquidity under stress, the first recommendation of Group of Thirty (2021) was that the Federal Reserve should create a standing repo facility (SRF) that provides very broad access to repo financing for US Treasury securities on adequate terms. The SRF that the Federal Reserve subsequently created in 2021 did not provide such access. This could have been in part due to concerns about creating moral hazards that would increase systemic risks, as broader SRF access may incentivize firms to increase their leverage (Group of Thirty 2022). The G30 have suggested that this moral hazard could be mitigated by centrally clearing repos provided by the SRF. Our investigation indicates that this agency problem may be counteracted when CCP risk management agency problems are mitigated effectively.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper. The Securities and Exchange Commission (SEC) disclaims responsibility for any private publication or statement of any SEC employee or Commissioner. This paper expresses the authors' views and does not necessarily reflect those of the Commission, its staff members or Commissioners.

---

[33] Group of Thirty (2021) emphasizes that concerns about the concentration of risk at CCPs and their transparency and governance should be addressed properly.

ACKNOWLEDGEMENTS

R. Cont's research was supported by Royal Society APEX Award AX160182, "Mathematical modelling of systemic risk". We are grateful for comments and advice from Robert Anderson, Richard Berner, Darrell Duffie, David Murphy, Simon Potter, Oliver Richard, Manmohan Singh, Nizar Touzi and Jessica Wachter. For discussions and comments, we also thank Cindy Alexander, Jasmine Boatner, Marcus Burnett, Kevin Chen, Xiaowei Ding, Juan Echeverri, Kay Giesecke, Lisa Goldberg, Ulrich Karl, Y. C. Loon, Joshua Mallett, Natan Misak, Luis Molestina Vivar, Lars Tyge Nielsen, Martin Ockler, Markus Pelger, Anders Ryden, Leon Tatevossian and Bruce Tuckman, as well as participants at QuantMinds 2023 and seminar participants at the Bank of England, the European Central Bank, Columbia University, New York University, the SEC, University California, Berkeley, the International Association for Quantitative Finance and Stanford University.

REFERENCES

Andersen, L., and Dickinson, A. (2018). Funding and credit risk with locally elliptical portfolio processes: an application to CCPs. Working Paper, Social Science Research Network (https://doi.org/10.2139/ssrn.3161154).

Baranova, Y., Holbrook, E., MacDonald, D., Rawstorne, W., Vause, N., and Waddington, G. (2023). The potential impact of broader central clearing on dealer balance sheet capacity: a case study of UK gilt and gilt repo markets. Staff Working Paper 1026, Bank of England, London (https://doi.org/10.2139/ssrn.4488256).

Basel Committee on Banking Supervision (2023). Capital requirements for bank exposures to central counterparties. Standards Document, Bank for International Settlements, Basel. URL: www.bis.org/basel_framework/chapter/CRE/54.htm.

Bernanke, B. S. (2011). Clearinghouses, financial stability, and financial reform. Speech, 2011 Financial Markets Conference, April 4, Stone Mountain, Georgia. URL: www.bis.org/review/r110405b.pdf.

Biais, B., Heider, F., and Hoerova, M. (2016). Risk-sharing or risk-taking? Counterparty risk, incentives, and margin. *Journal of Finance* **71**(4), 1669–1698 (https://doi.org/10.1111/jofi.12396).

Bolton, P., and Oehmke, M. (2015). Should derivatives be privileged in bankruptcy? *Journal of Finance* **70**, 2353–2393 (https://doi.org/10.1111/jofi.12201).

CCP12 (2023). 2022 Annual markets review in central counterparty clearing: a CCP12 report. Report, July, CCP12 Global, Asterdam. URL: https://ccp-global.org/sites/default/files/2025-02/CCP12_AMR_2022_20230713-final.pdf.

Choi, G., Ortega, F., and Singh, M. (2021). Emerging market securities access to global plumbing. Working Paper WP/21/094, International Monetary Fund, Washington, DC (https://doi.org/10.5089/9781513574165.001).

Clancy, L. (2023). Lessons on bank resolution, from Silicon Valley to Zurich. Risk.net, June 20. URL: www.risk.net/regulation/7956990/lessons-on-bank-resolution-from-silicon-valley-to-zurich.

Committee on Payments and Market Infrastructures–Board of the International Organization of Securities Commissions (2017). Resilience of cental counterparties: further guidance on the PFMI. Report, Bank for International Settlements, Basel. URL: www.bis.org/cpmi/publ/d163.pdf.

Committee on Payments and Market Infrastructures–Board of the International Organization of Securities Commissions (2022). Client clearing: access and portability. Report, Bank for International Settlements, Basel. URL: www.bis.org/cpmi/publ/d210.pdf.

Committee on Payment and Settlement Systems–Technical Committee of the International Organization of Securities Commissions (2012). Principles for financial market infrastructures. Report, Bank for International Settlements, Basel. URL: www.bis.org/cpmi/publ/d101a.pdf.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance* **1**(2), 223–236 (https://doi.org/10.1080/713665670).

Cont, R. (2015). The end of the waterfall: default resources of central counterparties. *Journal of Risk Management in Financial Institutions* **8**, 365–389 (https://doi.org/10.69554/JZQP9267).

Cont, R., and Kan, Y. H. (2011). Statistical modeling of credit default swap portfolios. Working Paper, Social Science Research Network (https://doi.org/10.2139/ssrn.1771862).

Cont, R., and Kokholm, T. (2014). Central clearing of OTC derivatives: bilateral vs multilateral netting. *Statistics and Risk Modeling* **31**, 3–22 (https://doi.org/10.1515/strm-2013-1161).

Cont, R., Deguest, R., and He, X. (2013). Loss-based risk measures. *Statistics and Risk Modeling* **30**(2), 133–167 (https://doi.org/10.1524/strm.2013.1132).

Dudley, W. (2014). Global financial stability: the road ahead. Remarks at the Tenth Asia-Pacific High-Level Meeting on Banking Supervision, Auckland, New Zealand. URL: www.newyorkfed.org/newsevents/speeches/2014/dud140226.

Duffie, D. (2015). Resolution of failing central counterparties. In *Making Failure Feasible: How Bankruptcy Reform Can End Too Big to Fail*, Jackson, T. H., and Scott, K. E. (eds), Chapter 4. Hoover Institution Press, Stanford, CA.

Duffie, D. (2020). Still the world's safe haven? Redesigning the US Treasury market after the COVID-19 crisis. Working Paper 62, Hutchins Center on Fiscal and Monetary Policy, Brookings Institution, Washington, DC. URL: www.brookings.edu/articles/still-the-worlds-safe-haven/.

Duffie, D., and Zhu, H. (2011). Does a central clearing counterparty reduce counterparty risk? *Review of Asset Pricing Studies* **1**(1), 74–95 (https://doi.org/10.1093/rapstu/rar001).

Duffie, D., Fleming, M., Keane, F., Nelson, C., Shachar, O., and Van Tassel, P. (2023). Dealer capacity and US Treasury market functionality. Staff Report 1070, Federal Reserve Bank of New York (https://doi.org/10.59576/sr.1070).

Embrechts, P., Kluppelberg, C., and Mikosch, T. (1997). *Modelling Extremal Events for Insurance and Finance*. Stochastic Modelling and Applied Probability, Volume 33 Springer (https://doi.org/10.1007/978-3-642-33483-2).

European Union (2013a). Commission Delegated Regulation (EU) No 153/2013 of 19 December 2012 supplementing Regulation (EU) No 648/2012 of the European Parliament and of the Council with regard to regulatory technical standards on requirements for central counterparties. *Official Journal of the European Union* **56**(L52), 41–74 (https://doi.org/10.3000/19770677.L_2013.052.eng).

European Union (2013b). Commission Delegated Regulation (EU) No 152/2013 of 19 December 2012 supplementing Regulation (EU) No 648/2012 of the European Parliament and of the Council with regard to regulatory technical standards on capital requirements for central counterparties. *Official Journal of the European Union* **56**(L52), 37–40 (https://doi.org/10.3000/19770677.L_2013.052.eng).

Group of Thirty (2021). US Treasury markets: steps toward increased resilience. Report, G30 Working Group on Treasury Market Liquidity. URL: https://stein.scholars.harvard.edu/sites/g/files/omnuum5951/files/stein/files/g30_paper.pdf.

Group of Thirty (2022). Status update, US Treasury markets: steps toward increased resilience. Report, G30 Leadership Team.

Garratt, R., and Zimmerman, P. (2015). Does central clearing reduce counterparty risk in realistic financial networks? Staff Report 717, Federal Reserve Bank of New York (https://doi.org/10.2139/ssrn.2646040).

Ghamami, S. (2015). Static models of central counterparty risk. *International Journal of Financial Engineering* **2**(2), Paper 1550011 (https://doi.org/10.1142/S2424786315500115).

Ghamami, S. (2020). The unintended impact of collateral on financial stability. Risk.net, 31 July. URL: www.risk.net/risk-management/7661131/the-unintended-impact-of-collateral-on-financial-stability.

Ghamami, S., and Glasserman, P. (2017). Does OTC derivatives reform incentivize central clearing? *Journal of Financial Intermediation* **32**, 76–87 (https://doi.org/10.1016/j.jfi.2017.05.007).

Ghamami, S., and Glasserman, P. (2019). Submodular risk allocation. *Management Science* **65**(10), 4656–4675 (https://doi.org/10.1287/mnsc.2018.3156).

Ghamami, S., Glasserman, P., and Young, P. (2021). Collateralized networks. *Management Science* **65**(10), 4656–4675 (https://doi.org/10.1287/mnsc.2018.3156).

Ghamami, S., Paddrik, M., and Simpson, Z. (2023). Central counterparty default waterfalls and systemic loss. *Journal of Financial and Quantitative Analysis* **58**(8), 3577–3612 (https://doi.org/10.1017/S0022109022001351).

Greenwood, R., Hanson, S. G., Stein, J. C., and Sunderam, A. (2017). Strengthening and streamlining bank capital regulation. *Brookings Papers on Economic Activitiy* **2017**(2), 479–565 (https://doi.org/10.1353/eca.2017.0020).

Hansmann, H. (2013). All firms are cooperatives – and so are governments. *Journal of Entrepreneural and Organizational Diversity* **2**(2), 1–10 (https://doi.org/10.5947/jeod.2013.007).

Hart, O., and Moore, J. (1996). The governance of exchanges: members' cooperatives versus outside ownership. *Oxford Review of Economic Policy* **12**(4), 53–69 (https://doi.org/10.1093/oxrep/12.4.53).

Huang, W. (2019). Central counterparty capitalization and misaligned incentives. Working Paper 767, Bank for International Settlements, Basel. URL: www.bis.org/publ/work767.htm.

Ingber, J. (2017). The development of the Government Securities Clearing Corporation. Federal Reserve Bank of New York, *Economic Policy Review* **23**(2), 33–50. URL: www.newyorkfed.org/research/epr/2017/epr_2017_gscc_ingber.

Ivanov, S. (2017). Initial margin estimations for credit default swap portfolios. *The Journal of Financial Market Infrastructures* **5**(4), 23–49 (https://doi.org/10.21314/JFMI.2017.079).

JPMorgan Chase (2020). A path forward for CCP resilience, recovery and resolution. Press Release, March 10. URL: www.jpmorgan.com/insights/markets/markets/a-path-forward-for-ccp-resilience-recovery-and-resolution.

Kuong, J. C.-F., and Maurin, V. (2023). The design of a central counterparty. *Journal of Financial and Quantitative Analysis* **59**(3), 1257–1299 (https://doi.org/10.1017/S0022109023000121).

McLaughlin, D. (2018). Skin in the game. *The Journal of Financial Market Infrastructures* **7**(1), 47–55 (https://doi.org/10.21314/JFMI.2018.101).

McNeil, A. J., Frey, R., and Embrechts, P. (2015). *Quantitative Risk Management*. Princeton University Press.

Modigliani, F., and Miller, M. H. (1958). The cost of capital, corporation finance, and the theory of investment. *American Economic Review* **48**(3), 261–297. URL: www.jstor.org/stable/1809766.

Murphy, D. (2017). I've got you under my skin: large central counterparty financial resources and the incentives they create. *The Journal of Financial Market Infrastructures* **5**(3), 57–74 (https://doi.org/10.21314/JFMI.2017.073).

Thiruchelvam, S. (2022). BoE official signals tough stance on CCP skin in the game. Risk.net, November 11. URL: https://bit.ly/3G8xgD3.

Tsay, R. S. (2010). *Analysis of Financial Time Series*. Wiley (https://doi.org/10.1002/9780470644560).

Tucker, P. (2014). Are clearinghouses the new central banks? Keynote Speech, April 11, Federal Reserve Bank of Chicago Annual Symposium on Central Clearing, Harvard Kennedy School, Cambridge, MA.

Walker, J. (2023). CCP "skin in the game" still dwarfed by member contributions. Risk.net, February 27. URL: www.risk.net/risk-quantum/7956011/ccp-skin-in-the-game-still-dwarfed-by-member-contributions.

APPENDIX TO "SKIN IN THE GAME: RISK ANALYSIS OF CENTRAL COUNTERPARTIES" BY RAMA CONT AND SAMIM GHAMAMI

A.1 Proof of Proposition 4.3

Note that

$$P(L_i^1 > x) = P\left(U_1 > M_1 + D_1 + S + \frac{x(D - D_1)}{D_i}\right). \tag{A.1}$$

Since $D_i = c_i D$, we can write

$$P(L_i^1 > x) = P\left(U_1 > M_1 + D_1 + S + \frac{x(1 - c_1)}{c_i}\right).$$

Set $A \equiv D_1 + S + x(1 - c_1)/c_i$. The probability on the right side above can be written as

$$P(U_1 - M_1 > A) = qP(U_1 - M_1 > A | U_1 > M_1), \tag{A.2}$$

where $q = P(U_1 > M_1)$. Recall that $(U_1 - M_1)|U_1 > M_1 \sim \text{Pa}(\alpha, \kappa_1)$, and so

$$P(U_1 - M_1 > A | U_1 > M_1) = \left(\frac{\kappa_1 + A}{\kappa_1}\right)^{-\alpha}. \tag{A.3}$$

We now remove the dependence of the term on the right side above on κ_1. To do so, note that

$$q_D = P(U_1 > M_1 + D) = qP\left(U_1 > M_1 + D \mid U_1 > M_1\right).$$

Again, using our modeling assumption $(U_1 - M_1)|U_1 > M_1 \sim \text{Pa}(\alpha, \kappa_1)$, we can write

$$P\left(U_1 > M_1 + D | U_1 > M_1\right) = \left(\frac{\kappa_1 + D}{\kappa_1}\right)^{-\alpha},$$

and so

$$\frac{D}{\kappa_1} = \left(\frac{q}{q_D}\right)^{1/\alpha} - 1. \tag{A.4}$$

We note that (A.2)–(A.4) give (4.4). This completes the proof. □

A.2 Proof of Proposition 4.4

The proof is similar to the proof of Proposition 4.3; we only need to modify the last part of the proof of Proposition 4.3 as follows. Note that

$$P(U_j > M_j + E_j) = q_D = qP(U_j > M_j + E_j | U_j > M_j) = q\left(\frac{\kappa_j + E_j}{\kappa_j}\right)^{-\alpha}.$$

This gives

$$\left(\frac{q}{q_D}\right)^{1/\alpha} - 1 = \frac{E_j}{\kappa_j}, \quad \text{which leads to } \kappa_j = \frac{E_j}{(q/q_D)^{1/\alpha} - 1}.$$

This leads to (4.6) as its right side does not depend on κ_j. □

A.3 Proof of Proposition A 4

We use the following lemma to prove Proposition A 4.

LEMMA A 1 *Under Assumption 4.1, we have*

$$E[(U_1 - M_1 - W)^+] = \frac{q\kappa_1}{\alpha - 1}\left(\frac{\kappa_1 + W}{\kappa_1}\right)^{-\alpha+1}. \tag{A.5}$$

where $W > 0$ *is a constant.*

PROOF OF LEMMA A 1 Note that

$$E[(U_1 - M_1 - W)^+] = E[(U_1 - M_1)1_A] - WP(A) \tag{A.6}$$

where 1_A is the indicator of the event $A = \{U_1 - M_1 > W\}$. First, consider the expectation on the right side above. The conditional probability density function of $(U_1 - M_1)|U_1 > M_1$ is

$$f_1(u) = \frac{\alpha}{\kappa_1}\left(\frac{\kappa_1 + u}{\kappa_1}\right)^{-\alpha-1},$$

where $u \geqslant 0$. Note that

$$E[(U_1 - M_1)1_A] = qE[(U_1 - M_1)1_A \mid U_1 > M_1].$$

We use integration by parts to calculate the conditional expectation above to derive

$$E[(U_1 - M_1)1_A] = q\int_W^\infty uf_1(u)\,\mathrm{d}u = q\left(\frac{\alpha W + \kappa_1}{\alpha - 1}\right)\left(\frac{W + \kappa_1}{\kappa_1}\right)^{-\alpha}. \tag{A.7}$$

Next, consider the last term on the right side of (A.6) and note that

$$P(U_1 - M_1 > W) = q\left(\frac{\kappa_1 + W}{\kappa_1}\right)^{-\alpha}. \tag{A.8}$$

It is straightforward to see that (A.7) and (A.8) give (A.5). This completes the proof. □

A.3.1 Proof of Proposition A 4

Using Assumption 4.1, we can write

$$P(U_i - M_i > E_i) = q_D = q\left(\frac{\kappa_i + E_i}{\kappa_i}\right)^{-\alpha},$$

and so we have

$$\kappa_i = \frac{E_i}{(q/q_D)^{1/\alpha} - 1}.$$

Lemma A 1 and the above expression for κ_i give

$$\sum_{i=1}^{N} E[U_i - M_i - D_i]^+ = \frac{Eq[1 + c_1(q/q_D)^{1/\alpha} - 1)]^{-\alpha+1}}{(\alpha - 1)[(q/q_D)^{1/\alpha} - 1]}, \tag{A.9}$$

where $E = \sum_{i=1}^{N} E_i$. Given our formulations of S and $\tilde{S}$ in (4.2) and (4.11), we have derived (5.1). Dividing the right side of (5.1) by the right side of (A.9) gives (A.30). This completes the proof.

A.4 Proof of Proposition 6.1

Note that

$$P(\check{L}_i^j > x) = P\left(U_j - M_j > \check{S} + x\frac{M - M_j}{M_i}\right), \tag{A.10}$$

for any $x \geqslant 0$ and $i \neq j$. Now, consider the more typical multilayered waterfall (in the presence of a separate prefunded default fund, where the IM pool is not mutualized). Conditional on the default of member j, the probability distribution of losses to member i's default fund contribution can be written as

$$P(L_i^j > x) = P\left(U_j - M_j > D_j + S + x\frac{D - D_j}{D_i}\right).$$

Clearly, when U_j are drawn from the same distribution under both waterfall structures, setting $x = 0$ and $S = \check{S}$ gives (6.4). This completes the first part of the proof. We use the following Lemma for the second part.

LEMMA A 2 *If $U_i/\sigma_i \sim T(0, \nu)$ has a mean-zero Student-t distribution with $\nu > 1$ degrees of freedom then*

$$\frac{E_i}{\sum_{j=1}^{N} E_j} = \frac{M_i}{\sum_{j=1}^{N} M_j}, \tag{A.11}$$

where either VaR or ES is used in calculating M_i and E_i.

It is not difficult to see that Lemma A 2 gives $M_i/(M-M_j) = D_i/(D-D_j)$. Consequently, (6.5) holds for any $x > 0$ and $i \neq j$ as long as $D_i > 0$. This completes the proof.

PROOF OF LEMMA A 2 The proof uses standard results in the theory of quantitative risk management (McNeil *et al* 2015).

First, suppose that M_i and E_i are calculated based on VaR. That is, $M_i = \mathrm{VaR}_q(U_i)$ and $E_i = \mathrm{VaR}_{q_D}((U_i - M_i)^+)$ with $q_D < q \leqslant .01$. Given that $U_i/\sigma_i \sim T(0, \nu)$, it is straightforward to show that $M_i = \sigma_i t_q$, where t_q denotes the inverse of Student t cumulative distribution function with mean zero and degrees of freedom ν evaluated at $1 - q$. This results in

$$\frac{M_i}{\sum_{j=1}^N M_j} = \frac{\sigma_i}{\sum_{j=1}^N \sigma_j}. \tag{A.12}$$

Since $M_i = \sigma_i t_q$ and $U_i/\sigma_i \sim T(0, \nu)$, it is straightforward to show that $E_i = \sigma_i(t_{q_D} - t_q)$. Consequently,

$$\frac{E_i}{\sum_{j=1}^N E_j} = \frac{\sigma_i}{\sum_{j=1}^N \sigma_j}. \tag{A.13}$$

Second, suppose that M_i and E_i are calculated based on ES. When $U_i/\sigma_i \sim T(0, \nu)$, it is well-known and can be easily shown that,

$$M_i = \mathrm{ES}_q(U_i) = \sigma_i \frac{g(t_q)}{q} \left(\frac{\nu + t_q^2}{\nu - 1} \right),$$

where g denotes the Student t probability density function with degrees of freedom ν and mean zero. To calculate $E_i = \mathrm{ES}_{q_D}((U_i - M_i)^+)$, we use the following well-known result (McNeil *et al* 2015, Ch.2),

$$\mathrm{ES}_{q_D}((U_i - M_i)^+) = \mathrm{ES}_{q_D}(U_i) - M_i,$$

to derive

$$\mathrm{ES}_{q_D}((U_i - M_i)^+)) = \sigma_i \left(\frac{g(t_{q_D})}{q_D} \left(\frac{\nu + t_{q_D}^2}{\nu - 1} \right) - \frac{g(t_q)}{q} \left(\frac{\nu + t_q^2}{\nu - 1} \right) \right),$$

So, (A.12) and (A.13) hold under ES and Student t distribution. This completes the proof. □

A.5 Cover-n Case

We now extend our analysis to the scenario where the prefunded default fund is sized under the cover-n rule; $2 \leqslant n \leqslant N$. To simplify the notation, suppose that

$$E_N \leqslant E_{N-1} \leqslant \cdots \leqslant E_2 \leqslant E_1.$$

In what follows, when necessary, we append a subscript or superscript n to loss variables and other model components to differentiate the cover-n case from the cover-one analysis presented in the main body of the paper. Under the cover-n DF, we can write

$$D_{s,n} = \sum_{i=1}^{n} E_n.$$

Suppose that DF is allocated to members proportional to E_i, and member i's DF contribution is denoted by $D_{i,n}$.

As before, default losses from both member and the CCP's perspective are conditional on the default of a single member.[34] We note that under the cover-n DF, the basic results of Section 3 remain unchanged. Specifically, (3.6)-(3.9) and (3.16)-(3.19) continue to hold under the cover-n rule. However, while $P(L_0^1 > 0) = q_D$ under the cover-one DF, we have

$$P(L_{0,n}^1 > 0) = P(U_1 - M_1 > D_{s,n}) < q_D,$$

under the cover-n rule. Also, in the absence of any capital contributions by the CCP, we will have

$$P(\tilde{L}_{i,n}^1 > D_{i,n}) = P(U_1 - M_1 > D_{s,n}) < q_D,$$

under the cover-n rule.[35]

A.5.1 Pareto-based SITG: First Layer

Under Assumption 4.1, the distribution of $L_{i,n}^1$ can be expressed in terms of E_1. For the cover-n DF, we can derive

$$P(L_{i,n}^1 > x) = q\left[1 + \left(\left(\frac{q}{q_D}\right)^{1/\alpha} - 1\right)\left(\sum_{k=1}^{n} c_k + \frac{S_n}{E_1} + \frac{(1-c_1)}{c_i}\frac{x}{E_1}\right)\right]^{-\alpha}. \tag{A.14}$$

The proof is omitted as it is similar to the proof of Proposition 4.3.

Given (A.14), the target loss probability of $\pi_n = P(L_{i,n}^1 > 0)$, where $\pi_n \leqslant q_D$, results in the following SITG formulation

$$S_n = \left(\frac{(q/\pi_n)^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1} - \sum_{k=1}^{n} c_k\right) E_1. \tag{A.15}$$

---

[34] It is straightforward to carry out the analysis conditional on $n \geqslant 2$ simultaneous defaults.

[35] It is also straightforward to extend our analysis to derive *scenario-B SITG formulations* in this more general setting. That is, our framework could be used to construct ranges for the first and second layer SITG that satisfy a battery of incentive compatibility constraints.

Simple algebra gives the following second expression,

$$S_n = \left[\left(\frac{(q/\pi_n)^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1}\right)\left(\frac{c_1}{\sum_{k=1}^{n} c_k}\right) - c_1\right] D_{s,n}. \tag{A.16}$$

This formulation is useful as S_n is more explicitly written as a percentage of total DF. Recall that we have formulated SITG as a percentage of total DF in the main body of the paper in the cover-one case. We also note that setting $\pi_n = q_D$ gives

$$S_{q_D,n} = \left(1 - \sum_{k=1}^{n} c_k\right) E_1. \tag{A.17}$$

Conditional on the default of member $j \neq 1$ and under the cover-n rule, we can derive

$$P(L_{i,n}^{j} > x) = q\left[1 + \left(\left(\frac{q}{q_D}\right)^{1/\alpha} - 1\right)\left(\sum_{k=1}^{n} c_k + \frac{S_n}{E_j} + \frac{x}{E_j}\frac{(1-c_j)}{c_i}\right)\right]^{-\alpha}. \tag{A.18}$$

The proof is similar to the proof of Proposition 4.4 and so is omitted.

Comparing (A.14) and (A.18), we can write $P(L_{i,n}^{j} > x) \leqslant P(L_{i,n}^{1} > x)$. Consequently, formulating S_n according to (A.15) with the target loss probability $\pi_n \leqslant q_D$ gives

$$P(L_{0,n}^{j} > 0) \leqslant P(L_{i,n}^{j} > 0) \leqslant q_D \tag{A.19}$$

where $L_{0,n}^{j} = (U_j - M_j - D_{s,n})^{+}$. Note that

$$P(L_{i,n}^{j} > 0) \leqslant q_D,$$

is the basic and overarching ICC (3.9) we introduced in the cover-1 DF setting.

A.5.2 Pareto-based SITG: Second Layer

Conditional on the default of member 1, the probability that the loss of member i exceeds its default fund contributions becomes[36]

$$P(\tilde{L}_{i,n}^{1} > D_{i,n}) = q\left[1 + \left(\left(\frac{q}{q_D}\right)^{1/\alpha} - 1\right)\left(\frac{\sum_{k=1}^{n} c_k}{c_1} + \frac{S_n}{E_1} + \frac{\tilde{S}_n}{E_1}\right)\right]^{-\alpha}. \tag{A.20}$$

This is to be compared with the first part of Proposition 4.5 in the cover-one case. Note that with S_n being sized according to (A.15) and setting $\tilde{S}_n = 0$, we will have

[36] The proof is similar to the proof of Proposition and so is omitted.

$P(\tilde{L}^1_{i,n} > D_{i,n}) \leqslant \pi_n \leqslant q_D$. We denote this upper bound corresponding to $\tilde{S}_n = 0$ by $\tilde{\pi}_{0,n}$.

Given S_n as in (A.15), fixing the second target loss probability $\tilde{\pi}_n$, where $\tilde{\pi}_n \leqslant \tilde{\pi}_{0,n} \leqslant \pi_n \leqslant q_D$, and working backwards, we have

$$\tilde{S}_n = \left[\frac{(q/\tilde{\pi}_n)^{1/\alpha} - (q/\pi_n)^{1/\alpha}}{(q/q_D)^{1/\alpha} - 1} + \left(1 - \frac{1}{c_1}\right) \sum_{k=1}^{n} c_k \right] E_1. \tag{A.21}$$

Note that for the special case where $\pi_n = q_D$, $\tilde{S}_n$ becomes

$$\tilde{S}_n = \left[\frac{(q/\tilde{\pi}_n)^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1} + \left(1 - \frac{1}{c_1}\right) \sum_{k=1}^{n} c_k - 1 \right] E_1. \tag{A.22}$$

Similar to cover-one case, for a CCP that operates under the cover-n rule, our SITG formulations lead to

$$P(\tilde{L}^j_{i,n} > D_{i,n}) < P(L^j_{0,n} > 0) \leqslant P(L^j_{i,n} > 0) \leqslant q_D. \tag{A.23}$$

That is, our Pareto-based formulations of S_n and $\tilde{S}_n$ will lower members' default loss probabilities below q_D.

REMARK A 3 We can write (A.21) as

$$\tilde{S}_n = \left[\left(\frac{(q/\tilde{\pi}_n)^{1/\alpha} - (q/\pi_n)^{1/\alpha}}{(q/q_D)^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^{n} c_k} \right) + c_1 - 1 \right] D_{s,n}. \tag{A.24}$$

This formulation is useful as it explicitly expresses $\tilde{S}_n$ as a fraction of total prefunded default fund in the cover-n case. In short, it is insightful to compare $\tilde{S}_n$ formulated in (A.24) with $\tilde{S}$ formulated in (4.10) in the cover-one case. We also note that

$$S_n + \tilde{S}_n = \left[\left(\frac{(q/\tilde{\pi}_n)^{1/\alpha} - 1}{(q/q_D)^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^{n} c_k} \right) - 1 \right] D_{s,n}. \tag{A.25}$$

This is our proposed lower bound on minimum CCP regulatory capital requirements under cover-n DF. It is useful to compare this lower bound with the lower bound (5.1) derived under cover-one DF.

A.6 CCP Risk Capital

CCP risk capital refers to capital requirements for bank exposures to CCPs (Basel Committee on Banking Supervision 2023). Policymakers sometimes borrow ideas from bank capital regulation when regulating CCPs. For instance, in the context of credit and counterparty risk capital, a CCP is viewed as a financial firm holding portfolios of financial assets with N counterparties. The CCP's minimum risk-based capital requirement is then a percentage of its *risk weighted assets*, where the risk weights represent the credit quality (default probability) of members (counterparties) and assets represent the CCP's exposure to its members net IM and DF over a given time period, (Ghamami 2015). Adopting the classical bank capital regulation framework for CCP capital regulation can be seen in the Basel Committee's formulation of CCP risk capital. In our setting, the Basel Committee's *hypothetical* CCP capital requirement is formulated as

$$K_{ccp} = k_r \times \sum_{i=1}^{N} E[U_i - M_i - D_i]^+ \omega_i, \tag{A.26}$$

where the capital ratio k_r is set to 8%, and the minimum requirement for risk weights ω_i is equal to 20%. K_{ccp} with this fixed risk weight is then used to formulate the CCP risk capital rule. More specifically, the amount of capital that a bank needs to hold against its exposure to a CCP is an increasing function of K_{ccp}.

Since the sum of first and second layers of SITG can be viewed as a lower bound for CCP capital requirements, there are different ways where $S + \tilde{S}$ can be used to test the adequacy of CCP risk capital. The following approach will be insightful. Note that if the inequality

$$K_{ccp} \geqslant S + \tilde{S}, \tag{A.27}$$

does not hold, regulators may need to revisit the definition or formulation of K_{ccp}, adjust k_r or ω, or they can replace K_{ccp} with $S + \tilde{S}$. Since ω represents the average credit quality of clearing members, we have $0 \leqslant \omega \leqslant 1$. This suggests that $0 \leqslant \omega k_r \leqslant 1$. Consequently, if

$$S + \tilde{S} \geqslant \sum_{i=1}^{N} E[U_i - M_i - D_i]^+, \tag{A.28}$$

holds under our framework, policymakers can revisit CCP risk capital. That is, our framework implies that under (A.28), the CCP risk capital rule can underestimate bank exposures to CCPs. Now, consider the following ratio,

$$R = \frac{S + \tilde{S}}{\sum_{i=1}^{N} E[U_i - M_i - D_i]^+}. \tag{A.29}$$

Proposition A 4 derives a useful expression for R.

TABLE A.1 Lower bound R for the ratio of SITG to K_{ccp} for different values of α, c_1, q, q_D, $\tilde{\pi}_0$, and $\tilde{\pi}$. We note that $R > 1$ in most cases unless the concentration ratio is unrealistically small and $\tilde{\pi}$ is close to $\tilde{\pi}_0$, which gives $\tilde{S}$ close to zero. Rows in italics report the part of the parameter space producing $R < 1$.

R	α	c_1	q (bps)	q_D (bps)	$\tilde{\pi}_0$ (bps)	$\tilde{\pi}$ (bps)
4.19	2	.05	100	50	31	20
2.10	2	.05	100	50	31	30
8.56	2	.10	100	50	31	20
1.08	2	.10	80	70	63	60
4.56	2	.15	100	70	54	45
37.86	2	.20	100	50	33	10
7.66	2	.20	100	50	33	32
58.96	2	.30	100	50	34	10
18.83	3	.10	100	50	30	10
5.28	3	.10	100	50	30	29
39.6	3	.20	100	50	32	10
3.85	3	.20	70	60	50	53
.66	*3*	*.01*	*100*	*50*	*29*	*25*
16.6	4	.20	100	40	22	20
68.19	4	.40	100	20	9.69	9
4.81	4	.15	90	70	57	50
1.76	5	.01	100	50	27	10
.69	*5*	*.01*	*100*	*50*	*27*	*25*
11.19	5	.15	100	50	30	25
8.66	5	.15	100	50	30	29
.69	*6*	*.01*	*100*	*50*	*27*	*25*
4.31	6	.10	100	80	66	50
7.30	6	.10	100	50	29	25

PROPOSITION A 4 *Under Assumption 4.1, the lower bound (A.29) for the ratio of SITG to Basel CCP capital, K_{CCP}, is given by*

$$R = q^{-1} c_1 (\alpha - 1) \left[\left(\frac{q}{\tilde{\pi}} \right)^{1/\alpha} - \left(\frac{q}{q_D} \right)^{1/\alpha} \right] \left[1 + c_1 \left(\left(\frac{q}{q_D} \right)^{!1/\alpha} - 1 \right) \right]^{\alpha - 1}. \tag{A.30}$$

The following numerical example shows that condition (A.28) holds in most parts of the model parameter space.

EXAMPLE A 5 Table A.1 reports R in (A.30) for different values of the tail index α, concentration ratio c_1, loss probabilities associated with IM and DF confidence levels, q and q_D, and member target loss probabilities associated with unfunded DF,

$\tilde{\pi}$. As discussed earlier, to have $\tilde{S} > 0$, the target loss probability $\tilde{\pi}$ should satisfy $\tilde{\pi} < \tilde{\pi}_0$, where $\tilde{\pi}_0$ is defined in (4.9).[37] Note that R is an increasing function of c_1. It is clear that $R > 1$ in most parts of the parameter space unless the concentration ratio is very small and $\tilde{\pi}$ is close to $\tilde{\pi}_0$, giving values of $\tilde{S}$ close to zero.

In short, Proposition A 4 along with the above numerical example indicate that capital rules on banks due to their exposures to CCPs may not sufficiently absorb central clearing risks.

[37] Recall that $\tilde{\pi}_0$ is the loss probability associated with unfunded DF when we set $S = (1 - c_1)D$ and $\tilde{S} = 0$.